Precision Bevel Gears with Low Tooth Count

By Stephen P. Radzevich

An analysis focused on the geometry and the kinematics of right-angle bevel gears with low tooth count shows that inequality of base pitches of the gear and mating pinion is the root cause of insufficient performance.

RIGHT-ANGLE BEVEL GEARS ARE A PARTICULAR CASE of intersected-axis gearing (further I$_{g}$-gearing) with an arbitrary value of the shaft angle. Commonly, bevel gears with the base cone angle ($\Gamma_b$ for the bevel gear and $\gamma_b$ for the bevel pinion) larger than the root cone angle ($\Gamma_f$ for the bevel gear and $\gamma_f$ for the bevel pinion), that is, when the inequalities $\Gamma_b > \Gamma_f$ and $\gamma_b > \gamma_f$ are observed, are referred to as low-tooth-count (LTC) gears.

The geometry and the kinematics of gears that have 12 teeth or fewer are the main focus of this paper. All of the equations derived for LTC gears are valid for gears with an arbitrary tooth count — not only for gears with a large tooth count. When operating, right-angle bevel gears often generate vibration and produce excessive noise. Dynamic loading of the gear teeth can result in the tooth failure. These problems become more severe in bevel gearings with low tooth count. The performed analysis shows that inequality of base pitches of the gear and mating pinion is the root cause for insufficient performance of LTC gears.

In most applications, the main purpose of I$_{g}$-gearing is to smoothly transmit a rotation and torque between two intersected axes. Gear pairs that are capable of transmitting a uniform rotation from the driving shaft to the driven shaft are referred to as the geometrically accurate intersected-axis gear pairs (or, in other words, the ideal intersected-axis gear pairs).

Three requirements need to be fulfilled in order for a bevel gear pair to be referred to as the geometrically accurate bevel gear pair:

• The geometry of the tooth flanks of a bevel gear and a mating bevel pinion has to obey the condition of conjugacy. The condition of contact can be analytically represented in the form of dot product $n V_{\Sigma} = 0$ of the unit vector $n$ of a common perpendicular at the point of contact of tooth flanks $G$ and $P$ of the gear and the mating pinion and the vector of the velocity of the relative motion of the tooth flanks $G$ and $P$. The equation of contact, $n V_{\Sigma} = 0$, is commonly referred to as Shishkov’s equation of contact [1, 2]. This equation was proposed by Shishkov as early as 1948 (or even earlier).

• The geometry of the tooth flanks of a bevel gear and a mating bevel pinion has to obey the condition of conjugacy. To meet this requirement, common perpendicular at every point of contact of the tooth flanks $G$ and $P$ must intersect the axis of instant rotation (in other words, the pitch line). The shaft angle of a bevel gear pair is subdivided by the pitch line in a proportion that corresponds to gear ration of the bevel gear pair (see Equations 4 through 8).

• The geometry of the tooth flanks of a bevel gear and a mating bevel pinion has to ensure equal base pitches of (a) the gear, (b) the pinion, and (c) the operating base pitch of the gear pair. That is, these three base pitches must be equal to one another at every instant of time.

After the aforementioned requirements, which ideal bevel gearing has to obey, these considerations immediately follow.

First, the necessity to meet the condition of contact, $n V_{\Sigma} = 0$, is obvious. If the condition of contact is violated, this immediately results either in the interference of the tooth flanks $G$ and $P$ into each other or in departure of the tooth flanks $G$ and $P$ from one another. None of these two scenarios is valid in gearing.

Second, the condition of conjugacy of the tooth flanks $G$ and $P$ of the bevel gear and pinion is an equivalent to the well-known Willis’ theorem [3]. The Willis’ theorem relates to parallel-axis gears (or P$_{a}$-gears). No condition of conjugacy of the tooth flanks $G$ and $P$ in the cases of I$_{g}$-gearing as well as C$_{a}$-gearing (that is, for the case of crossed-axis gearing) is known so far. Below, the condition of conjugacy is enhanced to the case of I$_{g}$-gearing.

Third, it should be noted here that the cycle of meshing of only one pair of gear teeth is covered by the condition of conjugacy of the tooth flanks $G$ and $P$ of the bevel gear and pinion. Contact ratio in all gearings is always greater than one. Therefore, at a certain instant of time, two or more pairs of teeth are engaged in mesh simultaneously. To make the multiple contacts feasible, the equality of base pitches of (a) the gear, (b) the pinion, and (c) the operating base pitch is a must.

In Figure 5, the angular distance between each two adjacent desirable lines of contact ($LC_{3}^{\text{des}}$ and $LC_{1+1}^{\text{des}}$) is specified.
by the operating base pitch \( \phi_{\text{bop}} \) (Equation 19). Here, \( i \) is an integer number. The angle \( \phi_{\text{bop}} \) is measured within the plane of action, \( PA \), of the gear pair. This angle is centered at the common apex \( A_g = A_p = A_{pa} \) [4]. The operating base pitch, \( \phi_{\text{bop}} \), of a bevel gear pair is illustrated in Figure 5.

Equality of three base pitches, \( \phi_{bg} = \phi_{bp} = \phi_{\text{bop}} \), items (a) through (c), is referred to as the Fundamental Law of Gearing. Determination of the design parameters for ideal intersected-axis gearings is considered later in this paper.

**ELEMENTS OF THE KINEMATICS IN \( I_a \)-GEARING**

Consider the \( I_a \)-gear pair shown in Figure 1. The driving pinion is rotated about its axis, \( O_p \), with a certain angular velocity, \( \omega_p \). The driven gear is rotated about its axis, \( O_g \), with a certain angular velocity, \( \omega_g \). The axes \( O_g \) and \( O_p \) intersect at point \( A_{pa} \). These axes form an angle, \( \Sigma \). Commonly, this angle is equal to a right angle (that is, \( \Sigma = 90^\circ \)). However, \( I_a \)-gearings either with an angle, \( \Sigma > 90^\circ \), or with an angle, \( \Sigma < 90^\circ \), are also known. \( I_a \)-gearings can be internal and external. In a particular case, \( I_a \)-gearing degenerates to a so-called crown-gear-to-bevel-pinion gearing.

Rotations \( \omega_g \) and \( \omega_p \) can be interpreted as vectors, \( \omega_g \) and \( \omega_p \), correspondingly. The rotation vectors \( \omega_g \) and \( \omega_p \) are along the axes of rotation, \( O_g \) and \( O_p \). The actual directions of vectors \( \omega_g \) and \( \omega_p \) depend on the actual direction of the corresponding rotations, \( \omega_g \) and \( \omega_p \). The angle \( \Sigma \) is specified as the angle that is formed by the rotation vectors \( \omega_g \) and \( \omega_p \) that is:

\[
\Sigma = \angle(\omega_g, \omega_p)
\]

Equation 1

In relative motion, the resultant motion of the pinion with respect to the gear can be interpreted as a superposition of two rotations:

- The pinion is rotated, \( \omega_p \), about its axis, \( O_p \).
- The pinion is rotated about the gear axis, \( O_g \), with the rotation \(-\omega_g \).

The rotations \( \omega_p \) and \(-\omega_g \) are performed simultaneously. The resultant motion of the pinion is an instant rotation, \( \omega_{pl} \), about the pitch line, \( P_{ln} \) (Figure 1). The vector of instant rotation can be expressed in terms of the rotation vectors \( \omega_g \) and \( \omega_p \) as:

\[
\omega_{pl} = \omega_p - \omega_g
\]

Equation 2

The angle, \( \Sigma_{\text{pl}} \), formed by the rotation vectors \( \omega_g \) and \( \omega_{pl} \), is referred to as the gear pitch angle:

3 Note that rotations are not vectors in nature. Therefore, special care is required when treating rotations as vectors.
Equation 3 for the gear angle, $\Sigma_g$, casts to:

$$\Sigma_g = \tan^{-1}\left(\frac{\sin \Sigma}{\omega_p + \cos \Sigma}\right)$$  
Equation 4

For a shaft angle of 90°, Equation 4 reduces to:

$$\Sigma_g = \tan^{-1}\left(\frac{\omega_p}{\omega_g}\right)$$  
Equation 5

Similarly, the angle, $\Sigma_p$, formed by the rotation vectors $\omega_p$ and $\omega_{pl}$, is referred to as the pinion pitch angle:

$$\Sigma_p = \angle(\omega_{pl}, \omega_p)$$  
Equation 6

Equation 6 for the gear angle, $\Sigma_p$, casts to:

$$\Sigma_p = \tan^{-1}\left(\frac{\sin \Sigma}{\omega_p + \cos \Sigma}\right)$$  
Equation 7

Figure 2: Plane of action, PA, and the base cones in an orthogonal intersected-axis gearing [4]
For a shaft angle of 90°, Equation 7 reduces to:

\[ \Sigma_p = \tan^{-1}\left( \frac{\omega_g}{\omega_p} \right) \]  

Equation 8

It will be shown later that the pitch angles \( \Sigma_g \) and \( \Sigma_p \) are equal to pitch cone angles, \( \Gamma \) and \( \gamma \), of the gear and the pinion correspondingly; that is, the equalities \( \Sigma_g = \Gamma \) and \( \Sigma_p = \gamma \) are observed. With that said, one can proceed with the construction of the plane of action for an intersected-axis gear pair.

**PLANE OF ACTION AND BASE CONES IN \( I_a \)-GEARING**

In \( I_a \)-gearing, the plane of action, \( PA \), is a plane through the axis, \( P_{ln} \), of instant rotation (Figure 2). Two planes are used to construct the plane of action. A plane through the rotation vectors \( \omega_g \) and \( \omega_p \) is the first of them. This plane can be referred to as the axial plane of an \( I_a \)-gear pair. A plane through the rotation vector \( \omega_{pl} \) perpendicular to the axial plane is the second plane. This plane is referred to as the pitch plane of an \( I_a \)-gear pair. The plane of action, \( PA \), forms a transverse pressure angle, \( \phi_{t \omega} \), in relation to the pitch plane, \( PP \). The pressure angle, \( \phi_{t \omega} \), is measured within a plane, which is perpendicular to the vector of instant rotation, \( \omega_{pl} \).

The left upper portion of the schematic shown in Figure 2 is plotted within the plane of projections, \( \pi_1 \). The rest of two planes of projections, \( \pi_2 \) and \( \pi_3 \), of the standard set of the planes of projections, \( \pi_1 \pi_2 \pi_3 \), are not used in this particular consideration. Instead, two auxiliary planes of projections, namely the planes \( \pi_4 \) and \( \pi_5 \), are used. The axis of projections, \( \pi_1/\pi_4 \), is constructed so as to be perpendicular to the axis of instant rotation, \( P_{ln} \). The axis of projections, \( \pi_4/\pi_5 \), is constructed so as to be parallel to the trace of the plane of action, \( PA \), within the plane of projections, \( \pi_4 \).

When the gears rotate, the plane of action is rotated, \( \omega_{pa} \), about its axis, \( O_{pa} \). The axis \( O_{pa} \) is a straight line through the apex, \( A_{pa} \).
The axis $O_{pa}$ is perpendicular to the plane of action, $PA$. The rotation $\omega_{pa}$ is timed with the rotations $\omega_g$ and $\omega_p$ of the gear and the pinion. The rotation vector, $\omega_{pa}$, is along the axis $O_{pa}$.

Furthermore, a belt-and-pulley model can be constructed for the case of $I_a$-gearing. This concept is schematically illustrated in Figure 2. Two base cones are associated with the gear and the pinion of a geometrically accurate $I_a$-gear pair. The plane of action can be imagined as a flexible zero thickness film that is free to wrap on and unwrap from base cones of the gear and the pinion with no slippage. The plane of action is not allowed for any bending about an axis perpendicular to the plane, $PA$.

The base cones of the gear and the pinion are in tangency with the plane of action, $PA$. Therefore, each of them can be generated as an envelope to consecutive positions to the plane of action that is rotated about the gear (the pinion) axis of rotation. This makes it possible to derive an equation for the gear base cone angle, $\Gamma_b$:

$$\Gamma_b = \tan^{-1} \left( \frac{\sqrt{\sin^2 \varphi_g + \cos^2 \varphi_p \sin^2 \psi_{lw} \cos \varphi_{lw}}}{\cos \varphi_p \cos \varphi_{lw}} \right)$$

(A) Equation 9

A similar equation is valid for the pinion base cone angle, $\gamma_b$:

$$\gamma_b = \tan^{-1} \left( \frac{\sqrt{\sin^2 \varphi_g + \cos^2 \varphi_p \sin^2 \psi_{lw} \cos \varphi_{lw}}}{\cos \varphi_p \cos \varphi_{lw}} \right)$$

(B) Equation 10

The expressions for the calculation of the actual values of base cone angles (see Equations 9 and 10) show that the angles $\Gamma_b$ and $\gamma_b$ do not depend on the rotations of the gear $\omega_g$ and the pinion $\omega_p$, that is, the configuration of the pitch line, $P_{lw}$, in relation to the axes of rotations $O_g$ and $O_p$ is constant and does not vary when the gears rotate.

A contact perpendicular (a common perpendicular, in other terminology) through a point $K$ within the line of contact, $LC$, between the tooth flanks of the gear, $G$, and the mating pinion, $P$, can be constructed. In the geometrically accurate gearing, this perpendicular passes through the pitch line at every instant of time, and $P_i$ is
Field (Zone) of Action in \( I_a \)-Gearing

A bevel gear and a mating pinion interact with one another only within a portion of the plane of action, \( PA \), and not within the entire plane of action. This portion of the plane of action is commonly referred to as the field (zone) of action, \( ZA \). In \( I_a \)-gearing, the field (zone) of action is bounded by the lines of intersection of the plane of action by four boundary lines.

A circular arc of an outer radius, \( r_{opa} \), and a circular arc of a limiting radius, \( r_{pa} \), both centered at \( A_{pa} \), are the first two boundary lines of the field (zone) of action. The face width of the field (zone) of action, \( ZA \), is calculated as Equation 12 (see Figure 4).

\[
F_{pa} = R_{opa} - R_{pa}
\]  

Equation 12

Two straight lines of intersection, \( k_5l_5 \) and \( m_5n_5 \), of the plane of action, \( PA \), by the outer cones of the gear and the pinion are the second two boundary lines of the field (zone) of action (Figure 4).

Constructed in Figure 4, the angle \( \phi_z \) between the straight lines \( k_5l_5 \) and \( m_5n_5 \) is referred to as the angular width of the zone of action. The angular width, \( \phi_z \), of the zone of action is equivalent to the active portion, \( Z \), of the line of action, \( LA \), in \( I_a \)-gearing [5]. The angular width, \( \phi_{za} \), of the \( ZA \) can be expressed in terms of the design parameters of the gear and the mating pinion as:

\[
\phi_z = pr_{pa} \Gamma_a + pr_{pa} \gamma_a - \phi_{pa}
\]  

Equation 13

where:

- \( \Gamma_a \) is outer cone angle of the gear
- \( \gamma_a \) is outer cone angle of the pinion
- \( \phi_{pa} \) is projection onto the plane of action of the shaft angle \( \Sigma \), that is \( \phi_{pa} = pr_{pa} \Sigma \)

For reference purposes, the lines of tangency, \( c_5d_5 \) and \( a_5b_5 \), of the base cones of the gear and the pinion with the plane of action are shown in Figure 4. These straight tangent lines form an angle \( \phi_{pa} \), which is referred to as the total angular width of plane of action. It’s the desirable line of contact and tooth flank geometry in \( I_a \)-gearing. Interaction of the tooth flanks, \( G \) and \( P \), of a gear and a mating pinion in \( I_a \)-gearing takes place only within the zone of action. Therefore, the line of action, \( LC \), between the tooth flanks, \( G \) and \( P \), is always located within the \( ZA \). This gives an opportunity to a gear designer to ensure your longest possible tool life.
pick a planar curve of a reasonable geometry as the desired line of contact, \( LC_{\text{des}} \), for an \( I_a \)-gear pair. Use of a novel approach for the analytical description of the contact geometry of the tooth flanks, \( G \) and \( P \), of a gear and a mating pinion [4] enables one in determining the most favorable geometry of the desired line of contact, \( LC_{\text{des}} \). Let us assume that a desired line of contact between the tooth flanks, \( G \) and \( P \), of a gear and a mating pinion is given. For example, it could be shaped in the form of a circular arc that forms a desired spiral angle with a radial direction of the \( ZA \).

In a reference system, \( X_{pa}Y_{pa}Z_{pa} \), associated with the plane of action, the position vector of a point, \( \eta_{lc} \), of the line of contact, \( LC \), is a function only of one parameter. This could be a polar angle, \( \phi \), if the line of contact is specified in polar coordinates. With that said, the position vector of a point of the line of contact, \( \eta_{lc} \), can be presented as \( \eta_{lc} = \eta_{lc}(\phi) \).

When the gears rotate, the plane of action, \( PA \), rolls with no slippage over the base cone of the gear. In such a motion, the line of contact, \( LC \), travels (together with the \( PA \)) in relation to a referenced system, \( X_{g}Y_{g}Z_{g} \), associated with the gear. The parameter of the resultant motion of the line of contact, \( LC \), in relation to the referenced system, \( X_{g}Y_{g}Z_{g} \), is denoted by \( \theta_{g} \). For an analytical description of the transition from the reference system \( X_{pa}Y_{pa}Z_{pa} \) to the reference system \( X_{g}Y_{g}Z_{g} \), the position vector of a point of the line of contact, \( \eta_{lc} \), in the reference system \( X_{g}Y_{g}Z_{g} \), can be obtained using the relation

\[
\eta_{lc}(\theta_{g}) = \eta_{lc}(\phi) \left( \frac{\sin(\theta_{g})}{r_{lc}(\phi)} \right)
\]

where \( r_{lc} \) is the radius of the circular arc forming the desired line of contact.

Figure 5: Contact ratio in \( I_a \)-gearing [4]

Figure 6: Definition of tooth thickness, \( \phi_t \), and space width, \( \phi_w \), in \( I_a \)-gearing (measured within the pitch plane, \( PP \)) [4]
to the reference system \( X_{pa} Y_{pa} Z_{pa} \), operators of the resultant coordinate system transformation, \( R_{g} (pa \rightarrow g) \), can be composed \([4]\). Hence, this makes possible an expression:

\[
\mathbf{r}_{g}(\mathbf{q}, \mathbf{\theta}_{g}) = R_{s}(pa \rightarrow g) \mathbf{r}_{i+1}(\mathbf{q})
\]

**Equation 14**

for the position vector of a point, \( r_{g} \), of the gear tooth flank, \( G \). Similarly, an expression:

\[
\mathbf{r}_{p}(\mathbf{q}, \mathbf{\theta}_{p}) = R_{s}(pa \rightarrow p) \mathbf{r}_{i}(\mathbf{q})
\]

**Equation 15**

for the position vector of a point, \( r_{p} \), of the pinion tooth flank, \( P \), can be derived. Here, \( \theta_{p} \) denotes the parameter of the resultant motion of the line of contact, \( LC \), in relation to the referenced system, \( X_{pa} Y_{pa} Z_{pa} \), associated with the pinion, and the linear operator of the transition from the reference system \( X_{pa} Y_{pa} Z_{pa} \) to the reference system \( X_{pa} Y_{pa} Z_{pa} \), is designated as \( R_{s} (pa \rightarrow p) \). See Reference \([4]\) for more details on Equations 14 and 15.

**CONTACT RATIO IN \( I_{a} \)-GEARING**

The contact ratio shows the average number of pairs of teeth engaged in mesh simultaneously. Use of the contact ratio enables one in calculating the total length of the lines of contact in \( I_{a} \)-gearing.

The total contact ratio, \( m_{t} \), in \( I_{a} \)-gearing equals to the sum of the transverse, \( m_{p} \), and face, \( m_{f} \), contact ratios of the gear pair, that is:

\[
m_{t} = m_{p} + m_{f}
\]

**Equation 16**

The transverse (or profile) contact ratio, \( m_{p} \), can be specified as the ratio of the angular width of the zone of action, \( \phi_{pa} \), to the operating base pitch, \( \phi_{bop} \), of the bevel gear pair (see Figure 5):

\[
m_{p} = \frac{\phi_{pa}}{\phi_{bop}}
\]

**Equation 17**

The face contact ratio, \( m_{f} \), can be calculated as shown in Equation 18 (see Figure 5):

\[
m_{f} = \frac{\phi_{adv}}{\phi_{bop}}
\]

**Equation 18**

The central angle over which the line of contact spans is denoted by \( \theta_{adv} \) (Figure 5). This angle is referred to as the line of contact advanced angle. The actual value of the line of contact advanced angle depends on the geometry of gear teeth in their lengthwise direction. In the case of straight bevel gears, the line of contact advanced angle is zero. For the rest of \( I_{a} \)-gearings, the line of contact advanced angle is either positive (\( \theta_{adv} > 0^\circ \)) or negative (\( \theta_{adv} < 0^\circ \)).

The operating base pitch angle, \( \phi_{bop} \), is measured within the plane of action, \( PA \). This is the central angle between two corresponding points within the lines of contact for two adjacent pairs of teeth. For example, in Figure 5, the angle \( \phi_{bop} \) is shown between two points \( N_{g} \) and \( N_{p} \) that are located within a circular arc of an arbitrary radius \( r_{ypa} \) (centered at the apex \( A_{pa} \)) and the lines of contact \( LC_{i} \) and \( LC_{i+1} \) for two adjacent pairs of teeth. The operating base pitch angle can be calculated from:

\[
\phi_{bop} = 2\pi N_{g} \sin \Gamma_{b}
\]

\[
= 2\pi N_{p} \sin \gamma_{b}
\]

**Equation 19**

where:

- \( N_{g} \), \( N_{p} \) are tooth counts of the gear and the pinion correspondingly.

**TOOTH PROPORTIONS IN \( I_{a} \)-GEARING**

The discussed results of the study of \( I_{a} \)-gearing enable one to calculate tooth proportions in a geometrically accurate gear and pinion.

Tooth thickness, space width, and backlash are convenient to specify within the pitch plane, \( PP \), of a gear pair. When a gear pair operates, rotations of: (a) the gear; (b) the pinion; and (c) the pitch plane are synchronized with one another. Therefore, when a gear and a mating pinion turn through one tooth, the pitch plane also turns through one tooth, that is, the \( PP \) turns through an angle \( \phi_{N} \). The angle \( \phi_{N} \) is calculated as shown in equation 20 (see Figure 6):

\[
\phi_{N} = 2\pi N_{g} \sin \Gamma
\]

\[
= 2\pi N_{p} \sin \gamma
\]

**Equation 20**

where:

- \( \Gamma \) is pitch cone angle of the gear
- \( \gamma \) is pitch cone angle of the mating pinion

Once the angle \( \phi_{N} \) is determined, the angular tooth thickness, \( \phi_{t} \), and the angular space width, \( \phi_{w} \), can be calculated. By definition, the following equality is valid:

\[
\phi_{N} = \phi_{t} + \phi_{w}
\]

**Equation 21**

When designing a pinion, it is common to set the angular tooth thickness equal to the angular space width, that is:
\[ \varphi_t = \varphi_w = 0.5 \varphi_N \]  

Equation 22

When designing a gear, the gear tooth thickness is decreased by backlash, \( \varphi_B \), that is

\[ \varphi_B = \varphi_t - \varphi_w \]  

Equation 23

Other proportions among the design parameters \( \varphi_t \), \( \varphi_w \), and \( \varphi_B \), can be observed as well.

It should be stressed here again that there is no slippage between the pitch cones of the gear/pinion and the pitch plane when the gears rotate. Therefore, it can be imagined that the pitch plane wraps on or unwraps from the corresponding pitch cones. Because of this, the design parameters measured within the pitch plane correspond to the arc (not to the chordal) design parameters of the gear and the pinion.

Addendum and dedendum of a bevel gear also can be specified as the angular addendum and the angular dedendum of the gear. The angular tooth addendum in \( I_a \) gearing is specified by the angular distance between the pitch cone of the gear and the gear top-land cone (outer cone) of the gear. For bevel gears with standard tooth proportions, the tooth height of a bevel gear is set equal to module, \( m \). This makes it possible to calculate the angular addendum, \( \Gamma_a \), of the gear from the expression:

\[ \Gamma_a = \sin^{-1} \left( \frac{m}{r_{\text{opp}}} \right) \]  

Equation 24

In a similar manner, the angular dedendum is specified. For bevel gears with standard tooth proportions, the dedendum is greater than the addendum at a clearance, \( c \). Therefore, the angular dedendum, \( \Gamma_d \), of the gear is calculated as follows:

\[ \Gamma_d = \sin^{-1} \left( \frac{m + c}{r_{\text{opp}}} \right) \]  

Equation 25

The angular addendum, \( \Gamma_a \), and the angular dedendum, \( \Gamma_d \), of the gear tooth together specify the angular tooth height, \( \Gamma_t \), of the gear (see Figure 1):

\[ \Gamma_t = \Gamma_a + \Gamma_d \]  

Equation 26

Formulas similar to those aforementioned:

\[ \gamma_a = \sin^{-1} \left( \frac{m}{r_{\text{opp}}} \right) \]  

Equation 27

\[ \gamma_d = \sin^{-1} \left( \frac{m + c}{r_{\text{opp}}} \right) \]  

Equation 28

\[ \gamma_h = \gamma_a + \gamma_d \]  

Equation 29

are valid for the calculation of the angular addendum, \( \gamma_a \), and the angular dedendum, \( \gamma_d \), as well as the angular tooth height, \( \gamma_h \), of a standard bevel pinion (Figure 1). The aforementioned design parameters in intersected-axis gearing correlate to corresponding design parameters in parallel-axis gearing.

**CONCLUSION**

In an approach for designing geometrically accurate (ideal) bevel gearing, three issues are critical to achieve this goal.

First, the condition of contact, \( n V_\Sigma = 0 \), between the tooth flanks \( G \) and \( P \) of the gear and the mating pinion needs to be fulfilled. The condition of contact represented in the form of a dot product, \( n V_\Sigma = 0 \), is commonly referred to as the Shishkov’s equation of contact.

Second, geometrically accurate intersected-axis gearing must obey the condition of conjugacy. For this purpose, a new theorem is formulated for the case of \( I_a \)-gearing. This theorem is an equivalent of the well-known Willis’ theorem that is valid only for \( P_a \)-gearing. The discussed approach shows how to design bevel gears that meet the requirements imposed by the condition of conjugacy.

Third, as two or more pairs of teeth can be engaged in mesh simultaneously, geometrically accurate intersected-axis gearing must obey the fundamental law of gearing. This means that the triple equality must be fulfilled:

- Base pitch of the gear = Base pitch of the pinion = The operating base pitch
- The consideration in this paper is focused mainly on right-angle low-tooth-count bevel gearing. However, the reported results of the research are applicable for bevel gearings with different shaft angles and tooth counts.
- It should be noted that intersected-axis LTC gearing deserves more attention for numerous reasons. Inevitably, broader application of LTC gears in the future is one of the reasons. High-power-density gear trains are needed in the use of LTC gearing. All the gearings are evolving toward the highest possible power density being transmitted. This entails a broader application of LTC gearing in the future.

**REFERENCES**


**ABOUT THE AUTHOR:** Dr. Stephen P. Radzevich is a professor of mechanical engineering and a professor of manufacturing engineering. He received his M.Sc. in 1976, Ph.D. in 1982, and Dr. (Eng) Sc. in 1991, all in mechanical engineering. Radzevich has extensive industrial experience in gear design and manufacture. He has developed numerous software packages dealing with computer-aided design (CAD) and computer-aided machining (CAM) of precise gear finishing for a variety of industrial sponsors. His main research interest is the kinematic geometry of part surface generation, particularly with a focus on precision gear design, high-power-density gear trains, torque share in multiflow gear trains, design of special purpose gear cutting/finishing tools, and design and machine (finish) of precision gears for low-noise and noiseless transmissions of cars and light trucks. Radzevich has spent about 40 years developing software, hardware, and other processes for gear design and optimization. He trains engineering students at universities and gear engineers in companies. He has authored and co-authored over 30 monographs and handbooks and about 300 scientific papers, and he holds about 250 patents on inventions in the field.