TOOTHTIPS

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Preliminary results of testing of Low-tooth-count bevel gears of a novel design, part 1

THIS PAPER DEALS WITH BEVEL GEARS HAVING A *LOW-TOOTH-COUNT* (further, *LTC – gears*, for simplicity). In general sense, *LTC* bevel gears feature the *start-of-active-profile* cone angle, γ_l, smaller than the base cone angle, γ_b, of the gear. For *LTC* bevel gears the inequality $\gamma_l \leq \gamma_b$ is always observed. In a narrower sense, *LTC* bevel gears are viewed as those with 12 teeth and a fewer. Bevel gears with 12 teeth and a fewer are not thoroughly investigated yet, and the accuracy requirements for gearing of this particular type are covered neither by national, nor by international standards on gearing.[1]

Precision, that is, geometrically accurate (ideal) bevel gearing is the main focus of the paper in particular.

PREAMBLE

In order to be referred to as geometrically accurate, bevel gearing needs to meet the following three conditions all of which are of fundamental importance:

- **Condition of contact** is the first condition to be fulfilled in geometrically accurate bevel gearing. According to the condition of contact, at every point of contact, K, of tooth flanks of the gear, $\mathscr{G}$, and the pinion, $\mathscr{P}$, vector of the resultant relative motion, V_Σ, of the teeth flanks $\mathscr{G}$ and $\mathscr{P}$ is always locate in a common tangent plane to the surfaces $\mathscr{G}$ and $\mathscr{P}$ at K. Commonly, the condition of contact is specified by *Shishkov*'s equation of contact[2], $\mathbf{n} \bullet V_\Sigma = 0$, where $\mathbf{n}$ is the unit vector of a common perpendicular to the tooth flanks $\mathscr{G}$ and $\mathscr{P}$ at contact point K. Other forms of representation of the condition of contact are known as well.
- **Condition of conjugacy** of tooth flanks of the gear, $\mathscr{G}$, and the pinion, $\mathscr{P}$, is the second condition to be fulfilled in geometrically accurate bevel gearing. To fulfill the condition of conjugacy, a straight line along the common perpendicular, $\mathbf{n}$, at every point of contact, K, of tooth flanks of the gear, $\mathscr{G}$, and the pinion, $\mathscr{P}$, must intersect the pitch line, P_{ln}. Pitch line in bevel gearing is the axis of instant rotation of the pinion in relation to the gear (or of the gear in relation to the pinion when the pinion is considered stationary). More in detail the condition of conjugacy in bevel gearing is discussed in [1].
- **Equality of base pitches** of the gear, $\varphi_{b.g}$, and the pinion, $\varphi_{b.p}$, to the operating base pitch, $\varphi_{b.op}$, is the third condition to be fulfilled in geometrically accurate bevel gearing. In other words, at every point of contact, K, of tooth flanks of the gear, $\mathscr{G}$, and the pinion, $\mathscr{P}$, the identity $\varphi_{b.p} \equiv \varphi_{b.p} \equiv \varphi_{b.op}$ must be fulfilled. More in detail this condition in bevel gearing is discussed in [1].

A bevel gear pair is capable of transmitting a smooth rotation from the driving shaft to the driven shaft if and only if all of the above-listed three conditions are fulfilled. Bevel gears produced in the nowadays industry meet only the first condition and do not satisfy the second and the third conditions listed above. Therefore, all of them are the *approximate* gears.

DEVELOPMENT OF CAD MODEL FOR GEOMETRICALLY ACCURATE BEVEL GEARS

The development of a *CAD* model for geometrically accurate bevel gearing begins with the analytical description of the tooth flanks of the gear, $\mathscr{G}$, and the pinion, $\mathscr{P}$. A *belt-and-pulley* model of a bevel gear pair is helpful to develop an analytical description for the tooth flanks of the gear and the pinion.

2.1. A belt-and-pulley model for bevel gearing. Consider a right-angle bevel gear pair as an example. The tooth count of the gear is $N_g = 14$, the tooth count of the pinion is $N_p = 10$ and the transverse pressure angle $\phi_{t.w} = 22.5^\circ$.

Generation of the tooth flank of the gear, $\mathscr{G}$, is illustrated in Fig. 1. (Tooth flank, $\mathscr{P}$, of the mating pinion is illustrated in the similar manner.).

In Fig. 1, the gear and the pinion are rotated about their axes of rotation, O_g and O_p, accordingly. The axes O_g and O_p form a right angle with each other (that is, the shaft angle $\Sigma = 90^\circ$). Point of intersection of the axes

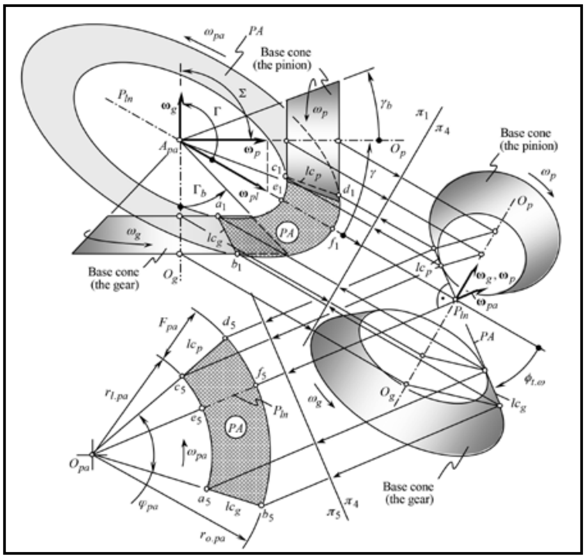

Fig. 1. Generation of the tooth flank of the geometrically accurate bevel gear, $\mathscr{P}$.

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As known, formulae and equations those derived for bevel gears are also valid for gears with a large tooth count and not vice versa.

This equation was proposed by Prof. Shishkov as early as in 1948 (or even earlier) in his paper: Shishkov, V.A., "Elements of Kinematics of Generating and Conjugating in Gearing", in: Theory and Calculation of Gears, Vol. 6, Leningrad: LONITOMASH, 1948. In detail, this equation is also discussed in the monograph: Shishkov, V.A., Generation of Surfaces in Continuously Indexing Methods of Surface Machining, Moscow, Mashgiz, 1951, 152 pages.

O_g and O_p is the common apex, A_{pa}, of the gear pair.

The rotation of the gear, ω_g, and the pinion, ω_p, are synchronized with one another.

Base cones are associated with the gear and the pinion. The base cone angle of the gear is designated as Γ_g, and the base cone angle of the pinion is designated as γ_b. The plane of action,*PA*, is tangent to the base cone of the gear and the pinion. lc_g is the line of tangency of the plane of action,*PA*, with the base cone of the gear. Similarly, lc_p is the line of tangency of the plane of action,*PA*, with the base cone of the pinion.

The plane of action forms the transverse pressure angle, $\phi_{t.w}$, with the plane through the common apex A_{pa} and that is perpendicular to the axial plane, that is, the plane through the axes O_g and O_p . The pitch line,P_{ln}, in a bevel gear pair is the line of intersection of the axial plane by the plane of action. The pitch line makes the gear pitch cone angle, Γ, with the axis of rotation of the gear, O_g. With the axis of rotation of the pinion, O_p , the pitch line makes the pinion pitch cone angle,γ.

If the rotations of the gear, ω_g, and the pinion, ω_p, are viewed as vectors $\boldsymbol{\omega}_g$ and $\boldsymbol{\omega}_p$ along the corresponding axis of rotation, O_g and O_p, then the instant rotation,ω_{pl}, of the pinion in relation the motionless gear is represented by the vector of instant rotation,$\boldsymbol{\omega}_{pl}$. Vector $\boldsymbol{\omega}_p$ is along the pitch line, P_{ln}.

When the gears rotate, the plane of action, *PA*, is unwrapped from the base cone of the pinion, and is wrapped on the base cone of the gear. Under such a scenario, the plane of action is viewed as a zero thickness film that is free for bending about the base cones and is absolutely rigid in all the rest of directions. The plane of action is rotated,ω_{pa}, about its axis,O_{pa}. The axis O_{pa} is through the common apex, A_{pa}, and is perpendicular to the plane of action, *PA*. Rotation of the plane of action, ω_{pa}, can also be represented in vector notation as the vector $\boldsymbol{\omega}_{pa}$ that is pointed along the axis O_{pa}.

An elementary analysis of Fig. 1 reveals that bevel gears of the proposed design are insensitive to the shaft angle alterations.

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