DESIGN OF A DOUBLE SPIRAL BEVEL GEARSET
With the development of CNC machines with 5 or more axes and additive manufacturing processes, the design of double spiral bevel gears becomes feasible with the possibility of being applied in a wide range of applications.

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It is known that bevel gears are used to transmit rotary motion and torque between intersected axes in case there is no possibility of using a parallel axis gearsets, which are of simpler manufacture. Their most common geometries are straight bevel gear, spiral bevel gear, and helical bevel gear (skew bevel gear), and all these types of gearsets subject the bearings to three types of force vectors, namely: axial, radial, and tangential loads that consequently influence their dimensioning. Therefore, any attempt to minimize radial and axial loads will lead to a more economical solution for the bearing’s sizes.

Thus, the present study aims the development of a conical gearset designed with a double spiral, expecting that radial and axial loads decrease and hence affects the design of power transmission units, reducing the stresses acting on the bearings and, consequently, their weights.

Similar geometry was proposed in the past (a herringbone face gear was manufactured by Citroen); however, the manufacturing processes of that time did not allow such geometry to be developed and applied on a large scale, since, in most cases, casting processes were used without a subsequent surface finishing process. Nevertheless, with the development of CNC machines with 5 or more axes and additive manufacturing processes, the design of double spiral bevel gears becomes feasible with the possibility of being applied in a wide range of applications.

1 INTRODUCTION

Spiral conical gears have the geometry and, consequently, the calculation of the teeth dependent substantially on the kinematics of the machine tools used in the machining process. The tradition of the manufacturers is so great that some conical gears are commercially known by the name of the machines that generated them — Gleason, Klingelnberg, Coniflex, and Oerlikon conical gear type, for instance.

Among bevel gears, spiral bevel gears bring benefits in their operation relative to straight bevel gears due to lower noise and smoother transmission. Thus, for the same level of load to be transmitted, spiral bevel gears may have smaller diameters when compared to the straight ones [1].

When the forces acting on a straight bevel gear are analyzed, it is observed that this set is subjected to three components of force, namely: radial \( W_r \), axial \( W_a \), and tangential \( W_t \) [2]. Figure 1 shows the force scheme quoted for a straight bevel gear.

When analyzing the forces involved in spiral bevel gears, according to [3] there are still the same three components: tangential, radial, and axial. These components subject the assembly to stresses and directly affect the dimensioning of the bearings that support the shafts. In this way, when minimizing such efforts, the load imposed on the bearing assembly decreases. This reduction must be applied on the axial and radial components due to the fact that the tangential component is directly responsible for the torque transmission. This reduction and/or annulment of the axial components, for example, can be seen in a double helical cylindrical gearset (also called herringbone gears).

With the development of CNC machining centers of 5 or more axes with programming packages for the manufacture of gears and the development of additive manufacturing machines, the geometry of the bevel gears becomes unlimited and this new gear patterns can be developed. Considering that such double helical geometry may cancel or minimize axial loading in double spiral cylindrical gears, the objective of this work is to initiate the development of a set of double-spiral bevel gears in order to attempt to minimize axial and radial loads, propose an initial geometry of the gear set and present the parameters that give rise
to such geometry. If possible, this minimization of load may directly affect the performance of the bearings, that is, the reduction of the working forces on the bearings, which may reduce their masses and sizes.

2 BACKGROUND

According to ISO 23509:2016, the axial force on the pinion drive side \((F_{ax1,D})\) is given by Equation 1. For the coast side transmission, the axial force \((F_{ax1,C})\) is given by Equation 2.

\[
F_{ax1,D} = \left( \tan \alpha_{ncD} \sin \delta_1 + \tan \beta_{m1} \cos \delta_1 \right) F_{mt1} \quad \text{Equation 1}
\]

\[
F_{ax1,C} = \left( \tan \alpha_{ncC} \sin \delta_1 - \tan \beta_{m1} \cos \delta_1 \right) F_{mt1} \quad \text{Equation 2}
\]

In Equations 1 and 2, \(\alpha_{nc}\) and \(\alpha_{ad}\) represent the generated pressure angle on coast side and drive side respectively; \(\delta_1\) is the pitch angle; \(\beta_{m1}\) the mean spiral angle, and \(F_{mt1}\) represents the tangential force \([N]\) on the pinion that is given by Equation 3:

\[
F_{mt1} = \frac{2000T1}{dm1} \quad \text{Equation 3}
\]

In which \(dm1\) \([mm]\) represents the mean pitch diameter of the pinion and \(T1\) \([N.m]\) the torque transmitted. For these equations, if the value of the force obtained is positive (+) this means that the axial force is directed away from pitch apex, as can be seen in Figure 2. If the force value is negative (-) it is known that force acts toward the pitch apex.

It is expected that the resulting axial force \((F_{ax1,R})\), given as the sum of \(F_{ax1,C}\) and \(F_{ax1,D}\) is minimized when compared to the axial force generated by the same pinion when only one helix hand is applied over its entire face width. Also, the value of the radial force must be checked once, so this radial force on the pinion will be the axial force on the gear.

3 METHODOLOGY

For this study, as a first analysis, it was decided to generate a conical pair geometry with unitary gear ratio, obtaining, therefore, identical pinion and gear and shaft angle at 90°.

Each conical gear, once it is a double spiral application, will be composed of two parts, each with an opposite helix hand. It was chosen to generate a pinion, starting in its inner pitch diameter with right hand helix (R.H. helix) and ending in its outer pitch diameter with left hand helix (L.H. helix). For this configuration a total face width of 40.00 mm was chosen with helix hand transition in the position of 20.00 mm of face width. In this way, each double spiral pinion consists of two spiral bevel gears with independent helix as shown in Figure 3. The gears are of the standard type (pitch and root apex in one point). Besides these parameters, 20 teeth were adopted in the pinion and gear and pressure angle at normal section 20.00°.

3.1 GEOMETRY PARAMETERS

The spiral bevel gear geometry generated in this paper was developed according to [3], method 0, with tip, pitch, and root apexes converging in one point. Although 5-axis CNC machines or additive manufacturing machines generate all types of profiles, a profile generated by a face milling (single indexing method) is considered as a baseline for the developing of the teeth lead and profile geometries. The teeth are generated with a cutter radius, \(r_c\) of 100 mm ([5], [6] and [7]).

Tooth size can be established either setting the Outer Pitch Diameter \((d_{oa})\) or the mean normal module \((m_{mn})\). In this article, the authors previously adjusted the outer pitch diameter with a value of 166.8480 mm in order to have a mean normal module of 6.00 mm (see the calculation for mean normal module in the set of equations at the end of this section).
Addendum and dedendum coefficients are 1.000 and 1.2500 respectively and root radius factor is set to 0.3 (in module).

To generate the double spiral bevel gear, two gears of the same data but different helix hand are built as can be seen in Figure 4.

Table 1 and Table 2 summarize the data according to each gear. Both gears are then cut into two halves, called the inner half (first half) and the outer half (second half), and the inner half of the right-hand bevel gear is merged with the outer half of the left-hand bevel gear, generating the double spiral bevel gear seen in Figure 5.

Figure 6 presents the section view of the double spiral bevel gear with the geometries measured, ensuring the gear blank geometry calculated according to ISO 23509-2016 matches with the geometry constructed. Outer pitch, tip, and root diameters; mean pitch, tip, and root diameters; outer, mean, and inner cone distances; as well as pitch, face, and root angles must be the same for the right hand, left hand, and double spiral bevel gears. It is important to note the outer parameters of the inner gear shall be the same as the inner parameters of the outer gear, which
are the same as the mean parameters of both the right- and left-hand spiral bevel gears.

Finally, Table 3 and Table 4 summarize the double spiral data necessary to carry out the forces analysis. Parameters with one quote (’) refer to the inner bevel gear (first half) and parameters with two quotes (”) refer to the outer bevel gear (second half). The results of the forces analysis of the double spiral bevel gear are compared with the forces analysis of one of the spiral bevel gears previously shown in Table 1 and Table 2, but with a face width of 40 mm.

The set of geometry parameters calculated for the forces analysis in this work are presented in Equations 4 to 19:

\[ u = \frac{z_1}{z_2} = \frac{20}{20} = 1 \]  
\[ \delta_1 = \arctan \left( \frac{\sin \Sigma}{\cos \Sigma + u} \right) = \arctan \left( \frac{\sin 90}{\cos 90 + 1} \right) = 45^\circ \]  
\[ R_{e1} = \frac{d_{e1}}{2 \sin \delta_2} = \frac{166.848}{2 \sin 45^\circ} = 117.98 \text{ mm} \]  
\[ R_m = R_{e2} - \frac{b_2}{2} = 97.98 \text{ mm} \]  
\[ d_{m1} = 2R_{m1} \sin \delta_1 = 138,564 \text{ mm} \]  
\[ m_{mn} = 2R_m \frac{\sin \delta_2 \cos \beta_m}{z_2} = 2 \times 97.98 \sin 45^\circ \cos 30^\circ = 6,0000 \text{ mm} \]  
\[ R_{l1} = R_{e2} - b_2 = 117.98 - 40 = 77.98 \text{ mm} \]  
\[ \delta_{a1} = \delta_1 + \delta_{a1} = 45^\circ + 3.5043^\circ = 48.50^\circ \]  
\[ R_m' = R_{e2} - \frac{3b_2}{4} = 87.98 \text{ mm} \]  
\[ d_{m1'} = 2R_m' \sin \delta_1 = 124,4217 \text{ mm} \]  
\[ \beta' = \arcsin \left( \frac{-R_m' \cos \delta_1 - R_m^2 + R_m'^2}{2R_m' \cos \delta_1} \right) = 26.8173^\circ \]  
\[ R_{m''} = R_{e2} - \frac{b_1}{4} = 107.98 \text{ mm} \]  
\[ d_{m1''} = 2R_m'' \sin \delta_1 = 152,7051 \text{ mm} \]  
\[ \beta'' = \arcsin \left( \frac{-R_m'' \cos \delta_1 - R_m^2 + R_m''^2}{2R_m'' \cos \delta_1} \right) = 33.3028^\circ \]

A prototype of the double spiral bevel gear was manufactured using PLA through Fused Filament Fabrication (FFF) by additive manufacturing printer and is shown in Figure 7.

### 3.2 FORCE ANALYSIS

Accordingly, with the parameters used in Section 3.1, the equation forces for axial and radial components were analyzed. According to ISO 23509: 2016, the axial force being transmitted in drive side is given by Equation 1 and coast side by Equation 2.

For this study, once the conical pinion is divided into two parts, it was considered that the first part transmits power on the drive side and the second half on the coast side, since this pinion has a bi-spiral geometry. Thus, the axial force on the drive side will be rewritten considering the characteristics of the first half of the pinion according to Equation 20 and Equation 21 for coast side. The tangential force calculation will be rewritten according to Equations 22 and 23 considering these characteristics.

\[ F_{ax1,0} = \left( \tan \alpha_n \frac{\sin \delta_1'}{\cos \beta_{m1}'} + \tan \beta_{m1} \cos \delta_1 \right) F_{m1}' \]  
\[ F_{ax1,c} = \left( \tan \alpha_n \frac{\sin \delta_1''}{\cos \beta_{m1}''} - \tan \beta_{m1} \cos \delta_1'' \right) F_{m1}'' \]  
\[ F_{m1}' = \frac{2000 T_1}{\alpha m'_{1}} \]  
\[ F_{m1}'' = \frac{2000 T_1}{\alpha m''_{1}} \]  

Again, the (’) quote refers to the characteristics of the first half of the pinion and the (”) quote refers to the characteristics of the second half of the pinion.

For the tangential forces, the torque transmitted (T1) in the first and second half is the same, but the mean pitch diameters are different. Also, the tangential force will be calculated as a function of this transmitted torque.

To obtain the values from Equations 20 and 21, the parameters used are summarized in Table 5.

For radial forces, the equations for drive and coast side are given by Equations 24 and 25 [3]:

\[ F_{rad1,0} = \left( \tan \alpha_n \frac{\cos \delta_1}{\cos \beta_{m1}'} - \tan \beta_{m1} \sin \delta_1 \right) F_{m1}' \]  
\[ F_{ax1,c} = \left( \tan \alpha_n \frac{\cos \delta_1''}{\cos \beta_{m1}''} + \tan \beta_{m1} \sin \delta_1'' \right) F_{m1}'' \]

The results from these calculations are presented in Table 6.

As a comparative parameter, the resultant between axial and radial force was calculated once that both act on the same plane. This resultant force (F_{ar}) is given by Equation 26:

\[ F_{ar} = \sqrt{F_{ax1,0}^2 + F_{rad1,0}^2} \]

That gives a resultant force of \(12,2683 \times T_1\) N on the axial-radial plane.

### 4 DISCUSSION AND FUTURE WORKS

The results indicate that the axial and radial forces are more balanced in comparison with a single spiral bevel gear and the attempt to minimize the axial loads using a double spiral gear, as it happens in double helical or herringbone gears, as well as to minimize radial loads, is quite reasonable. The axial force on the pinion drive side is \(10.3810 \times T_1\) N, and the resultant of the whole pinion is minimized to \(8.3300 \times T_1\) N due to the axial force on the coast side that has a magnitude of minus-\(2.05 \times T_1\) N.

When radial forces are calculated for this double spiral bevel gear,
a value of minus 1.1103 \times T_1 N for the drive side and 10.1170 \times T_1 N for the coast side are obtained; therefore, the resultant radial force raises to 9.0068 \times T_1 N.

However, for the sake of comparison with a single spiral bevel gear, the resultant force on the axial-radial plane was calculated with a value of 12.2683 \times T_1 N. For the single spiral right hand (or left hand) bevel gear, with 40.00 mm face width and the same inner and outer pitch diameter this resultant force for axial-radial plane is 20.6494 \times T_1 N which is 68.9% higher than the double spiral bevel gear.

It is also important to note that the manufacturing of a double spiral bevel gears set is more complex in comparison with a single spiral bevel gears set. Moreover, this type of gear is not machinable using traditional bevel gear machinery such as Gleason or Klingelnberg machines, so this study became feasible because of the evolution of novel manufacturing processes such as the CNC machine centers with multi-axis and additive manufacturing. A prototype was made with AM. However, a gear manufactured by CNC machine center with multi-axis is going to be carried out subsequently.

This article takes into consideration the load analysis of a double spiral bevel gear in which the mean spiral angle was kept the same as for a single helix bevel gear with 40.00 mm face width spiral angles, and the width of each side is the same. Thus, optimization methods can be applied to evaluate which parameters (point of transition between helix hand, face width, spiral angles) could be modified in order to evaluate the impact of selected variables in the force calculation in an attempt to minimize the magnitude of the axial and radial vector loads. Besides that, double spiral bevel gears with tip, pitch, and root apex not in one point; and bevel gears with uniform depth might be investigated in future works.

![Figure 7: Prototype fabricated using FFF (fused filament fabrication).](image)

### Table 5: Parameters for force calculation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generated pressure angle on drive side</td>
<td>( \alpha_{n0} )</td>
<td>20,0000º</td>
</tr>
<tr>
<td>Generated pressure angle on coast side</td>
<td>( \alpha_{nc} )</td>
<td>20,0000º</td>
</tr>
<tr>
<td>Pinion first half pitch angle</td>
<td>( \delta_{b1} )</td>
<td>45,000º</td>
</tr>
<tr>
<td>Pinion second half pitch angle</td>
<td>( \delta_{b2} )</td>
<td>45,000º</td>
</tr>
<tr>
<td>Pinion first half mean spiral angle</td>
<td>( \beta_{m1} )</td>
<td>26,817º</td>
</tr>
<tr>
<td>Pinion second half mean spiral angle</td>
<td>( \beta_{m2} )</td>
<td>33,302º</td>
</tr>
<tr>
<td>Mean pitch diameter of the pinion first half</td>
<td>( dm_{1} )</td>
<td>124,421º mm</td>
</tr>
<tr>
<td>Mean pitch diameter of the pinion second half</td>
<td>( dm_{2} )</td>
<td>152,7051 mm</td>
</tr>
<tr>
<td>Tangential force on pinion first half</td>
<td>( F_{mat1} )</td>
<td>18,0737\times T_1 N</td>
</tr>
<tr>
<td>Tangential force on pinion second half</td>
<td>( F_{mat2} )</td>
<td>13,0971\times T_1 N</td>
</tr>
</tbody>
</table>

### Table 6: Force calculation results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial force on pinion drive side</td>
<td>( F_{ax1,D} )</td>
<td>10,3810\times T_1 N</td>
</tr>
<tr>
<td>Axial force on pinion coast side</td>
<td>( F_{ax1,C} )</td>
<td>-2,0510\times T_1 N</td>
</tr>
<tr>
<td>Resultant axial force on pinion</td>
<td>( F_{ax1,R} )</td>
<td>8,3300\times T_1 N</td>
</tr>
<tr>
<td>Radial force on pinion drive side</td>
<td>( F_{rad1,D} )</td>
<td>-1,1103\times T_1 N</td>
</tr>
<tr>
<td>Radial force on pinion coast side</td>
<td>( F_{rad1,C} )</td>
<td>10,1170\times T_1 N</td>
</tr>
<tr>
<td>Resultant radial force on pinion</td>
<td>( F_{rad1,R} )</td>
<td>9,0068\times T_1 N</td>
</tr>
</tbody>
</table>

### 5 CONCLUSIONS

The following conclusions can be drawn from this preliminary design of a double spiral bevel gear:

- The geometry of the blank can be designed as the same as a spiral bevel gear with a single spiral hand. The baseline used in this article is for a bevel gear with tip, pitch, and apex in one point and the geometry parameters of both gears equal.
- Due to discontinuity of the spiral direction, a double spiral bevel gear can be manufactured only by means of additive manufacturing machines (a prototype made in PLA was built) or multi-axis CNC machine centers. Machines that generate teeth by face hobbing (continuous hobbing process) and face milling (single hobbing process) are not able to manufacture such gears as the opposite helix hand teeth will be damaged by the cutter.
- There are other types of bevel gear designs to be investigated such as the influence of spirals with different widths, the mean spiral angle, or other identified parameters on optimization of the double spiral bevel gear designs.

### BIBLIOGRAPHY