LOAD INTENSITY DISTRIBUTION FACTOR EVALUATION FROM STRAIN GAUGES AT THE GEAR ROOT
Presenting a methodology to measure gear root strain distribution, as well as an analytical method to convert the measurements into flank load distribution.

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train gauges are commonly used to obtain the load intensity distribution on the flank of a gear mesh. The standard methodology consists in installing a set of gauges at the tooth root and measuring the strain distribution there. In order to get the load distribution on the flank, the strain data must be processed and changed into load intensity distribution on the tooth flank.

1 INTRODUCTION

Gear misalignment is one of the most important factors influencing gear fatigue loading. Due to high stiffness of the gear mesh a relatively small misalignment may result in a significant increase of specific loading, which may lead to premature gear failures.

Many techniques are used in industry to ensure good operational mesh alignment. From all known methods, direct tooth root strain measurements across the gear face-width offers the most accurate assessment of load intensity distribution.

The standard IEC 61400 — Part 4 establishes the design requirements for wind-turbine gearboxes and is widely used in the wind-turbine industry for certification purposes. From the 2012 edition, it is compulsory to perform the measurement of gear face load distribution at each load step using tooth root strain gauges (section 8.3.3. Workshop testing of a prototype gearbox). However, this standard does not provide any clarification on how these measurements should be performed.

The objective of this article is to describe the methodology developed by Gamesa Energy Transmission together with instrumentation partner JRD Transmission Dynamics to evaluate the gear mesh load intensity distribution from strain gauge measurements at the gear root. This methodology has been developed with wind-turbine gearboxes, but the authors believe it is also applicable to other highly loaded gear systems in other industries.

The experimental methodology described in this article is supported by different analytical studies. Section 3 describes the analytical work performed to establish the location of the strain gauges. Section 4 describes the data processing of measured signals, and section 5 describes the conversion from measured strain distribution to analytically derived load distribution on the gear flank.

The final objective of this article is to show an experimental methodology for the assessment of the load intensity distribution factor.

2 SYSTEM OVERVIEW

2.1 BACKGROUND

In an idealized gear pair, load intensity distribution across the face-width is uniform. With the relatively high-mesh stiffness, a relatively small misalignment will result in a significant change of load intensity reducing the effective face width and increasing specific gear load.

The ISO 6336-1 deals with the effect of gear alignment errors by the introduction of a longitudinal load distribution factor $K_{HF}$, which for the surface stress is defined as:

$$K_{HF} = \frac{\text{Maximum load per unit facewidth}}{\text{Average load per unit facewidth}} = \frac{\frac{F_b}{b}}{\frac{T_m}{b}} = \frac{\frac{F_b}{b}}{\frac{F_t}{b} \cdot K_T}$$

where

- $F$ is the flank load.
- $b$ is the gear flank width.
- $F_m$ is the mean transverse tangential load at the reference circle.
- $F_t$ is the transverse load.
- $K_A$ is the application factor.
- $K_v$ is the dynamic factor.

In practice, contact stress $\sigma_H$ and bending stress $\sigma_F$ depend on several $K$ factors and are described as follows [3]:

$$\sigma_H = Z_{BD} \cdot H_0 \cdot \sqrt{K_A \cdot K_V \cdot K_{HF} \cdot K_{HA}} \leq \sigma_{HP}$$

$$\sigma_F = F_0 \cdot K_A \cdot K_V \cdot K_{FB} \cdot K_{FA} \leq \sigma_{FP}$$

where

- $K_{HF}, K_{FB}$ are the face load distribution factors (contact stress and bending stress).
- $K_{HA}, K_{FA}$ are the transverse load factors (contact stress and bending stress).
- $H_0$ is the nominal contact stress.
- $F_0$ is the nominal tooth root stress.
- $\sigma_{HP}, \sigma_{FP}$ is the permissible stress (contact and bending).
- $Z_{BD}$ is the single pair tooth contact of the pinion/wheel.
It can be seen that for a given pair, gear stress (and hence gear life) critically depends on load distribution across the face, which is strongly influenced by gear misalignment.

This article describes the methodology to derive $K_{EB}$ from strain gauge measurements. Section 4 describes the data processing of the strain gauge signals to obtain the distribution of strains along the gear roots. The authors have chosen to name this strain distribution as $K_{EB}$.

$K_{EB}$ is the strain root distribution factor.

$$K_{EB} = \frac{\text{maximum strain per unit facewidth}}{\text{average strain per unit facewidth}} = \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{avg}}/b}$$  \hspace{1cm} \text{Equation 4}

where

- $\varepsilon$ is the tooth root strain.
- $b$ is the gear flank width.

### 2.2 GEARBOX/GEAR STAGE DESCRIPTION

For the present study, all the discussions are based on a first epicyclic stage of a wind-turbine gearbox of rated electrical power of 3.3Mw with 29.5tons total gearbox weight and an input speed of 10.53 rpm. Table 1 shows the general description of this gear stage.

### 2.3 MEASUREMENT SYSTEM OVERVIEW

An electronic system with strain gauges has to be installed to measure strain distribution across the gear-face width. A suitable number of strain gauges have to be placed along the face width. Section 3 discusses the number and positioning of strain gauges.

Based on high-frequency time domain strain gauge readings of each gauge, the peak-to-peak voltage corresponding to each mesh engagement is extracted and results are produced in the form of strain distribution $K_{EB}$ (see definition in section 2.1).

Figures 1 to 5 show details of the strain gauges system installation.

### 2.4 TELEMETER SYSTEM ELECTRONICS

The system uses specially developed gear alignment electronic modules developed by JRD Transmission Dynamics. The modules are equipped with 8-channel synchronous acquisition, strain gauge conditioning, and low-noise 12-bit ADC converter. The number of modules to be used depends on the gear size and number of gears to be measured.

The gear alignment module acquires simultaneous data from up to 8 strain gauges installed along the width of a gear root, at up to 14 kHz per channel (112 kHz aggregate data rate). Captured data can be analyzed using bespoke-supplied software or exported for external evaluation.

All modules are equipped with a digital Bluetooth system for wireless data transfer and communication. Data transfer is performed sequentially through serial communication to a host computer.

### 2.5 DATA ACQUISITION

Acquisition at each telemeter module follows these steps: After signal conditioning in analog stage, a digital circuit converts analog signals for all 8 channels to a digital stream during defined acquisition time and stores it on board RAM memory. On user demand, this data can be downloaded to a host computer. After downloading data for one module, another module can be activated so a new acquisition can...
Data acquisition is a sequential process; data is acquired and transmitted to the host computer module by module. Each strain gauge is connected in quarter bridge configuration. This configuration was chosen considering the limitation in the number of channels that can be measured simultaneously and the negligible effect of temperature in this particular application. Oil sump temperatures are usually below 60°C, and strain gauge readings are taken once the gearbox is under stable thermal conditions.

Data acquisition is carried out through proprietary software developed by the instrumentation partner. Software allows the definition by the user of the data acquisition parameters: sampling frequency and number of samples (acquisition time). These parameters should be chosen carefully. The sampling frequency must be high enough to be able to analyze each mesh event with enough samples. For planetary stage gears, a minimum of 30 points per mesh event is sought, which typically results in sampling frequency in the range of 2,000Hz to 4,000Hz. On the other hand, the number of samples must be large enough to cover enough mesh events; 8 to 10 revolutions of the input planet carrier have been found to give satisfactory results.

3 STRAIN GAUGES POSITIONING

3.1 INTRODUCTION
All the instrumentation strategy described in this document is supported by a different analysis oriented to answer the next three questions:

1. Where on the tooth root profile are the gauges installed?
2. How many gauges are on the flank?
3. Which component to instrument is on an epicyclical gear stage?

3.1.1 WHERE ON THE TOOTH ROOT PROFILE ARE THE GAUGES INSTALLED?
The gear tooth acts as a cantilever beam where a transverse load is applied and which originates tensile traction and compression stresses that are greatest in the reviewed section marked by 30-degree tangent according to ISO 6336. The strain will be the highest at these points; however, technical difficulties and functional ones (interlock of tooth head on gears) show that it’s not recommended to install gauges in these areas. Additionally, the voltage gradient near the max traction point must be considered, as the installation of the gauges becomes critical. A small positioning error may lead to higher measurement errors.

Therefore, the gauges are installed at the bottom of the tooth (see Figure 6) where the deformations are not maximum, but they are large enough to take readings and do not present as much variability as in the critical region of the 30-degree tangent. In section 3.2, there is an analytical study for supporting the earlier comment.

It is important to indicate that a gauge tensile reading will be...
followed by another compression reading because of the gear-mesh succession. Installing the strain gauges in the center plane between the teeth gives similar sensitivity to tensile and compression strain. The mesh cycle generates a peak-to-peak reading that gives greater accuracy, mitigating some possible error caused by installing a gauge not aligned with the root of the tooth.

### 3.1.2 HOW MANY GAUGES ARE ON THE FLANK?

The number of gauges to be installed in each root is decided by establishing a compromise between accuracy required, the space available, and the time and cost of installation. For the typical gear dimensions and $b/m_n$ ratios used in wind-turbine gearboxes, a number of 8 gauges spaced across the gear width has been found to produce satisfactory results.

A commonly produced gear flank microgeometry follows the next diagram (see Figure 7).

According to the authors' experience, at least one gauge should be installed on the end relief area (bER) to accurately measure the pressure release on these regions. The rest of the strain gauges are equally spaced to cover the remaining gear face width.

### 3.1.3 WHICH COMPONENT TO INSTRUMENT IS ON AN EPICYCICAL GEAR STAGE?

The analysis of the load distribution in planetary stages is divided into the load distribution of the mesh between the ring gear and the planets and the load distribution of the mesh of the sun gear with the planets.

There are two alternative possibilities (for epicyclical gear stage):

- **a.** Install strain gauges in the sun and ring gears (each strain gauge signal has the corresponding mesh event only).
- **b.** Install strain gauges in the planet gears (both mesh events are present in the same strain gauge signal).

See section 3.3 for a further discussion about the convenience of instrumenting a specific gear component.

### 3.2 INFLUENCE OF THE STRAIN GAUGE POSITION ERROR AND ANGULAR POSITION ERROR IN THE GEAR ROOT STRAIN MEASUREMENT

As a complement to the measurement system/strategy definition and prior to installing any gauge, an FE study based on the gear components of the current example was done. Here the sun gear case is analyzed.

The FE method allows us to obtain the stress and strain distribution at the surface of a solid under defined load conditions. The body is meshed using solid elements, and their surfaces covered using membrane elements. These membrane elements work as a gauge system.

The real gauge installation has an associated error related to the location and orientation of the measurement point. The FE analysis is used to quantify this error.

Figure 8 shows a view of the finite element model of the sun gear component used as an example. Abaqus 6.14 software is used for modeling.
Main characteristics/assumptions of the model:

- Combination of solid elements + a cover of shell elements acting as a gauge system. Quality checks carried out for mesh dimensioning.
- Symmetry applied. Only one-third of the sun gear component is meshed since it is part of a planetary system with three planet gears.
- Mesh density in the tooth profile adjusted to have enough representative diameters for load introduction. Straight lines for load applications along the gear flank (spur gear).
- The shell elements are used for stress/strain reporting, taking advantage of the nature of these kinds of elements. Values are calculated according to the element coordinate system. See [8] for shell-element definitions. The elements are properly oriented for the analysis:
  - Axis 1: Tangential direction. It is also normal to the flank gear.
  - Axis 2: Axial direction of the sun gear.
  - Axis 3: Normal to the surface of the membrane element.

The results for Axis 3 are null because the membrane elements are not able to support normal loads, strains, or stresses. This improves the quality of the results because it avoids errors generated by the extrapolation from the Euler nodes of the solid elements.

### 3.2.2 GAUGE POSITIONING ERROR

As it was introduced in section 3.1.1, the highest strain in the root is defined by ISO 6336-1. The FEM model has been used for strain distribution analysis under a specific loading condition. Figure 9 (left side) shows the strain distribution color map.

Figure 9 (right) shows the strain difference between neighbor nodes on the tooth root mesh. In the graph, axial axis corresponds with the gear face width, and angle axis corresponds with the angular position in the tooth root (bottom node — 0 degrees, end of the tooth root — 1.3 degrees). Equation 5 is used to calculate strain difference between nodes:

\[
\Delta \varepsilon = \varepsilon_{i+1} - \varepsilon_i
\]

where

- \(\varepsilon_i\) strain at node \(i\) (bottom node).
- \(\varepsilon_{i+1}\) strain at node \(i+1\) (adjacent nodes to bottom node left and right).

According Figure 9 (right), the strain difference at the tooth root is lower than in the critical section.

For a better understanding of the previous statement, the error is plotted against axial position in \% for both the root and the critical section. A positioning error of ±1mm is assumed. The strain of the misplaced strain gauge is compared with the ideally located strain gauge. According to the results in Figure 10, the effect of a tangential positioning error in the critical section is 10 times larger than in the tooth root.

Note that, to obtain the previous picture, a complete mesh cycle is calculated, loading the gear at different diameters and computing of the same tooth root, both the traction and compression produced by the meshing evolution.

Additionally, the effect of an angular positioning error has been evaluated. The strain tensor at a specific node of the finite element model is described as follows:

\[
\varepsilon = \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} \\
\varepsilon_{12} & \varepsilon_{22}
\end{bmatrix}
\]

where:

- \(\varepsilon_{11}\) is the strain in the direction along the tooth root.
- \(\varepsilon_{22}\) is the strain on the direction perpendicular to the tooth.
- \(\varepsilon_{12}\) is the cross term.

The real gauge system uses only one gauge at each position, and it is oriented with respect to \(\varepsilon_{11}\). The value of \(\varepsilon_{22}\) is considered low because there are no loads applied in this direction, but the \(\varepsilon_{12}\) can be significant since it is related with the torsion generated in the gear component.

When the axis of the gauge is not adequately oriented according to the theoretical direction due to an angular positioning error, the reading of the gauge will be affected by the cross term of the strain tensor.

To evaluate this effect, a virtual rotation to the ideal strain tensor has been applied as follows:

\[
R = \begin{bmatrix}
\cos(\alpha) & \sin(\alpha) \\
\sin(-\alpha) & \cos(-\alpha)
\end{bmatrix}
\]

\[
\varepsilon' = \begin{bmatrix}
\varepsilon'_{11} & \varepsilon'_{12} \\
\varepsilon'_{12} & \varepsilon'_{22}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} \\
\varepsilon_{12} & \varepsilon_{22}
\end{bmatrix} \begin{bmatrix}
\cos(\alpha) & \sin(\alpha) \\
\sin(-\alpha) & \cos(-\alpha)
\end{bmatrix}
\]

where

- \(R\) is the rotation matrix.
- \(\alpha\) is the error angle.
- \(\varepsilon'\) is the rotated strain tensor.
- \(\varepsilon\) is the original strain tensor.
- \(\varepsilon'_{11}\) is the strain measured by the gauge with an angular positioning error.
For the analysis, an angular positioning error of 5 degrees has been assumed. The calculated error in the tooth root versus the calculated error in the critical section is represented in the Figure 11.

Figure 11 shows the critical section is more affected by an angular positioning error. Additionally, it must be noted gear components with less torsion under load will be more robust from an angular positioning error point of view.

Based on these analyses, the tooth root is selected as the most adequate location for strain gauge installation.

3.3 GEAR CONTACT EVOLUTION ON AN EPICYCLOIDAL GEAR STAGE.

Detailed analysis of the gear contacts has been done based on a ROMAX model of the wind-turbine gearbox under study on the present document. (Figure 12)

The virtual gearbox includes all those elements in the real gearbox — shafts, bearings, gears, and housings — which contribute in some way to the gear contacts misalignments from a stiffness distribution point of view.

The virtual model is used to predict the gear-mesh misalignment for each one of the gear contacts under a specific load case.

Figure 13 and Figure 14 show the different contact patterns predicted along a complete planet carrier rotation. Load per unit length is plotted and both views are from rotor side to generator side:

According to the previous results, special care must be taken when deciding which gear component to instrument. Since the ring gear is a static component, the information captured by an instrumented gear tooth in the ring gear is position related, that means that, depending which location you choose, you will have the corresponding strain distribution and it will always be the same for every planet passing. This is important whether these measurements are going to be used for microgeometry updating. The authors’ proposal would be to locate and instrument that gear tooth where the contact pattern is expected to be centered.

In the case of the sun or planet gear, the fact the contact moves along the face width related with the rotation is dealt with by acquiring enough mesh cycles to cover all this variability. Then the mean and worst-case distributions can be obtained.

Based on these analyses, the authors decided to install strain gauges in the planet gears.

The conclusion exposed is considered valid for the specific case of the studied wind-turbine gearbox. The boundary conditions of the gearbox, connection to surroundings, gravity, etc., affect directly to the way the system deforms and, therefore, to how the gear contacts behave. A dedicated study for other cases is strongly recommended.
To evaluate the gear-mesh intensity distribution, it is important to measure it at different load levels. The Gamesa Energy Transmission standard run in tests consists of six torque levels and can be used for strain-gauge measurements. Additionally, over-torque conditions are measured if possible. The effect of the running speed is considered negligible, so nominal speed is used for all measurements.

When the strain gauges are installed in the planets, two different meshing cycles can be observed in the time waveform; one corresponds to the mesh with the ring gear and the other to the mesh with the sun gear. In the mesh with the ring gear, the planet acts as a driven gear; therefore, during the start of the mesh event, the gauge first is under traction (positive signal) and then under compression (negative signal). In the mesh with the sun, the planet gear acts as a driver; therefore, the gauge first is under compression (negative signal) and then under traction (positive signal). This can be used to identify the ring and sun gear meshes.

Data is collected simultaneously in all 8 channels. Peak-to-peak values can be compared within individual mesh events to generate load distribution patterns of each mesh event. These mesh events are then superimposed. These results can be seen in Figures 16 and 17. The Astute® software provides an analysis tool for automatic detection of the peaks and calculation of the peak-to-peak values. However, this analysis also can be performed on non-proprietary software once the raw time domain strain gauge signals are available.

It can be observed that the mesh distribution is dynamically changing; after a sufficient number of revolutions, the average and worst-case mesh events can be found. For correlation with theoretical models and determining optimizations of micro geometry, the average mesh distribution is considered. This is achieved by averaging the peak-to-peak data of each individual strain gauge and calculating the mesh distribution of the averaged values. Figure 18 shows average strain mesh distributions for both ring and sun meshes. From these distributions, $K_{eb}$ factors are calculated according to the definitions in section 2.1.

The same procedure is repeated for the different torque levels. Figure 19 shows the different strain mesh distributions, and Figure 20 shows the corresponding $K_{eb}$ values.

### 5 $K_{eb}$ TO $K_{hb}$ CONVERSION METHOD

The previous sections have shown a method for the measurement of the strain distribution ($K_{eb}$) in the gear root when a gear is meshing under load. For $K_{hb}$, calculation is needed to translate this data to load distribution at the gear flank. A finite element model of the corresponding gear component is used to obtain the $k$-factors of the next equations that relates strain at the root with load distribution in the flank. See Figure 21.

$$e_i = \sum_{j=1}^{n} k_{ij} F_j$$

where

- $e_i$ is strain of i-gauge.
- $k_{ij}$ is stiffness relation between i-location and force applied at j-location.
- $F_j$ is force applied at j-location.
- $n$ is number of strain gauges.

Using the same finite element model, the linearity assumption is checked, loading the gear at different tooth diameters with uniform load. Figure 22 shows the strain distribution at the gear root for the
different load cases. Root 3 is the root loaded to traction, and Root 4 is the root loaded to compression.

As seen in Figure 22, the shape of the strain curves is quite similar; the main difference is the strain level. Based on this, the stiffness components on the stiffness matrix should be linear.

For the calculation of $K_{fj}$ factors, the finite element model is loaded at the highest point of single tooth contact with a known load distributed into a number of load points equal to the number of gauges.

The next figure shows the strain deformation (E11) at the nodes of the bottom diameter for each load case. This deformation is normal to the flank gear, and there is one curve for each load step.

In Figure 23, the continuous lines are results for the root under traction load, and the dashed lines are results for the root under compression. The combination of traction + compression per load case and per gauge position allows it to fill the stiffness matrix in Equation 10.

Based on a measured strain distribution and the matrix $[K]$, it is possible to solve Equation 10 to obtain the load distribution. Once this load distribution is known, $K_{Hb}$ is calculated according to Equation 1.

6 FUTURE WORK
Several interesting topics are identified for future research. Some of those include:

- Fully understanding of the strain gauge signal in no-mesh condition, i.e. effect of strain caused by hoop stresses in thin rim components (ring and planet).
- Correlation between dynamically changing gear mesh alignment and gearbox vibration.
- Measurements of load intensity distribution under varying load conditions from wind-turbine operation, i.e. emergency shutdown, low-wind conditions, active pitch control.
- Condition monitoring application of load-distribution measurements.

7 CONCLUSION
The standard IEC 61400 — Part 4 establishes the design requirements for wind-turbine gearboxes and is widely used in the wind-turbine industry for the certification purposes. From the 2012 edition, it is compulsory to perform the measurement of gear face
load distribution at each load step using tooth root strain gauges. However, this standard does not provide any clarification on how these measurements should be performed.

In this article, a methodology to measure gear root strain distribution ($K_{eb}$) has been described. An analytical method to convert these measurements into flank load distribution ($K_{fb}$) has been presented.

Additionally, the influence of strain-gauge position and angular errors in the root strain distribution measurement has been analyzed.

REFERENCES

[4] Use of Tooth Root Strain Gauge Results to Estimate Mesh Stiffness
[5] Electronic Gear Alignment Module for In-Service Gear Load Intensity Distribution Analysis

Figure 23: Strain output at bottom nodes with discrete loading.

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