UNDERSTANDING THE MOUNTING DISTANCE:
CROSSED-AXES GEARING (HYPOID GEARING)
In this article, a favorable configuration of bevel gears in crossed-axes gearing is discussed in which it is stressed that the mounting distance is one of the most important design parameters in crossed-axes gearing.

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Mounting distance is one of the most critical design parameters in crossed-axes gearing (in hypoid gearing, spiroid gearing, and so forth). In this article, the concept of the mounting distance in intersected-axes gearing [1] is evolved to the case of crossed-axes gearing. The mounting distance is discussed with a focus on “geometrically accurate crossed-axes gearing” (that is, with focus on “perfect” gearing). The term “geometrically accurate gearing” means gearing with zero deviations of the actual values of all the design parameters from their desired values. The disclosed results of the analysis are applicable to gears for crossed-axes gearing and those machined on the bevel gear generators available on the market.

INTRODUCTION

Crossed-axes gearing is extensively used in the nowadays industry. Two examples of application of crossed-axes gearing are illustrated in Figure 1. Application of a hypoid gear pair in the design of an automobile differential is shown in Figure 1a. A spiroid gearing is depicted in Figure 1b.

In order to ensure smooth running and favorable load distribution between gear teeth, gears for crossed-axes gearing have to be properly manufactured and accurately assembled in the housing. High power density (or, in other words, the “power-to-weight ratio”) in crossed-axes gearing along with quietness when operating is strongly desired. If engineered properly, gears in crossed-axes gearing can be installed in the same manner as spur or helical gears and behave and perform as well as spur and helical gears. To achieve these goals, certain requirements have to be fulfilled when designing, manufacturing, and assembling crossed-axes gear pairs. Accuracy of the mounting distance is one of the most critical design parameters in this regard.

1. ELEMENTS OF KINEMATICS AND GEOMETRY IN CROSSED-AXES GEARING

The concept of “geometrically accurate” crossed-axes gearing can be traced back to about 2008 when R—gearing has been proposed by Prof. S. Radzevich [2]. R—gearing is the only kind of crossed-axes gearing that features line contact between tooth flanks, $\mathcal{G}$ and $\mathcal{P}$, of a gear and a mating pinion. No other kinds of crossed-axes gearing with line contact between the tooth flanks, $\mathcal{G}$ and $\mathcal{P}$, are feasible at all.

A crossed-axes gear pair that is comprised of two spiral bevel gears is shown in Figure 2 as an example. When crossed-axes gearing operates, the gear spins, $\omega_g$, about its axis of rotation, $O_g$. The mating pinion spins, $\omega_p$, about its axis of rotation, $O_p$. The axes of rotation, $O_g$ and $O_p$, cross one another. The closest distance of approach of the axes of rotation, $O_g$ and $O_p$, is designated as, $C$. The center-distance, $C$, is a straight-line segment of the center-line, $\mathcal{C}$. The axes of rotation, $O_g$ and $O_p$, form a crossed-axes angle, $\Sigma$. The angle, $\Sigma$, is formed by two rotation vectors, $\omega_g$ and $\omega_p$ [that is, by definition $\Sigma = \angle(\omega_g, \omega_p)$].

The axis of rotation of the gear, $O_g$, is perpendicular to the center-line, $\mathcal{C}$, and intersects the center-line at the gear base-cone-apex, $A_g$ (see Figure 2). Similarly,
the axis of rotation of the pinion, \( O_p \), is also perpendicular to the center-line, \( \xi \), and intersects the center-line at the pinion base-cone apex, \( A_p \). Length of the straight-line segment, \( A_p A_g \), equals to the center-distance, \( C \), that is, an equality \( A_p A_g = C \) is valid.

The center-distance, \( C \), is subdivided by a point, \( A_{pa} \), onto two straight-line segments, \( A_g A_{pa} \) and \( A_p A_{pa} \). The ratio \( A_g A_{pa}/A_p A_{pa} \) equals to the gear ratio in the intersected-axes gear pair \( (A_g A_{pa}/A_p A_{pa} = u) \). Point, \( A_{pa} \), is the plane-of-action apex, \( A_{pa} \).

The axis of instant rotation, \( P_{in} \), is a straight line through the plane-of-action apex, \( A_{pa} \), perpendicular to the center-line, \( \xi \). The axis, \( P_{in} \), forms the angles, \( \Sigma_g \) and \( \Sigma_p \), with the axes of rotation, \( O_g \) and \( O_p \), correspondingly. The actual values of the angles, \( \Sigma_g \) and \( \Sigma_p \), can be expressed in terms of the gear ratio, \( u \), in the crossed-axes gear pair [2].

A base cone is associated with the gear. The gear base cone apex, \( A_g \), is located within the center-line, \( \xi \). Another base cone is associated with the pinion. The gear base cone apex, \( A_p \), is also located within the center-line, \( \xi \). The base cones of the gear and the mating pinion are tangent with the plane of action in the gear pair—this is a must for perfect performance of crossed-axes gearing [2].

Neither axial displacement of the gear nor axial displacement of the pinion is allowed from the position shown in Figure 2.

### 2. MOUNTING DISTANCE

To operate properly, gears in crossed-axes gearing have to be properly configured in relation to one another. Mounting distance is a key design parameter that helps keep gears configured properly, and it is the most important design parameter for ensuring correct operation. Mounting distance can be specified as the distance from a locating surface on the back of one gear (most commonly a bearing seat) to the gear base-cone apex, \( A_g \).

Theoretically, in a crossed-axes gear pair, a gear base cone apex, \( A_g \), a mating pinion base cone apex, \( A_p \), and the plane-of-action apex, \( A_{pa} \), have to be located within the center-line, \( \xi \), at distances, \( A_g A_{pa} \) and \( A_p A_{pa} \), as schematically illustrated in Figure 3.

In reality, in order to transmit power, gears need to be supported. A gear housing is used for this purpose. The gear housing features a corresponding number of bores that are used to support gear shafts in the housing. A centerline of a bore for the gear shaft is labeled as \( O_{h,g} \). Correspondingly, a centerline of a bore for the pinion shaft is designated by \( O_{h,p} \). The center-line, \( O_{h,g} \), intersects the center-line, \( \xi \), at point, \( A_{h,g} \). The straight line of axis, \( \xi \), at point, \( A_{h,g} \). Strong constraints on the gear configuration and its mating pinion in the gear housing are imposed by points, \( A_{h,g} \) and \( A_{h,p} \).

The gear has to be configured in the gear housing so as to keep the gear axis of rotation, \( O_g \), aligned with the axis, \( O_{h,g} \), in the gear housing (that is, \( O_g = O_{h,g} \)). The gear base cone apex, \( A_g \), has to coincide with the point, \( A_{h,g} \), of intersection of the centerlines, \( O_{h,g} \) and \( \xi \) (that is, \( A_g = A_{h,g} \)). Then, the rest of the important components, that is, bearings, shims, fasteners, and so forth, are put together so as to keep the gear mounting distance, \( MD_g \), to the blueprint. The gear mounting distance, \( MD_g \), is measured between the gear back face and the gear-base-cone apex, \( A_g \).

Similarly, the pinion must be configured in the gear housing to keep the pinion axis of rotation, \( O_p \), aligned with the axis, \( O_{h,p} \) (that is, \( O_p = O_{h,p} \)), in the gear housing. The pinion base cone apex, \( A_p \), has to coincide with the point, \( A_{h,p} \), of intersection of the centerlines, \( O_{h,p} \) and \( \xi \) (that is, \( A_p = A_{h,p} \)). Then, the rest of the important components, that is, bearings, shims, fasteners, and so forth, are put together to keep the pinion mounting distance, \( MD_p \), to the blueprint. The pinion mounting distance, \( MD_p \), is measured between the pinion back face.
Finally, once the gear and the pinion are properly mounted in the gear housing, their axes are aligned to the centerlines of the corresponding bores in the gear housing (that is, \(O_g = O_{hg.g}\) and \(O_p = O_{hp.p}\)), and the apexes, \(A_g\) and \(A_p\), are snapped together with the corresponding points, \(A_{h.g}\) and \(A_{h,p}\), as schematically illustrated in Figure 3.

An important conclusion can be drawn from the above discussion (see Figures 2 and 3):

**Intermediate conclusion:** In crossed-axes gearing, NO axial displacements of the gears from the position when the base cone apexes, \(A_g\) and \(A_p\), of a gear and a mating pinion coincide with the points of intersection, \(A_{h,g}\) and \(A_{h,p}\), of the centerlines, \(O_{h,g}\) and \(O_{h,p}\) in the gear housing by the center-line \(c\), are permissible.

Therefore, it is wrong practice to adjust for the backlash in crossed-axes gearing by means of “inward/outward” axial displacement of the gears. NO axial displacements of the gears are allowed.

In precision crossed-axes gearing, it is recommended to inspect the mounting distance at nominal operating load, and the pre-loaded bearings. A tolerance for the accuracy of the mounting distance is tight.

### 3. MOTIVATION

Consider a geometrically accurate crossed-axes gear pair \((C_{ab}—\text{gear pair})\). A rotation from a driving member to a driven member in the gear pair is transmitted through a line of contact, \(LC\), between the tooth flanks, \(\mathcal{G}\) and \(\mathcal{P}\), of the gear and the mating pinion.

Lines of contact, \(LC\), of various geometries are practically used in design of crossed-axes gear pairs. For simplicity, but without loss of generality, a straight line of contact \(ab\) is shown in Figure 4a. In reality, due to various factors that affect the real configuration of the gear and the pinion in relation to each other, the desirable line contact between the tooth flanks, \(\mathcal{G}\) and \(\mathcal{P}\), can be substituted with their “edge contact,” as illustrated in Figure 4b. When edge contact is observed, the tooth flanks, \(\mathcal{G}\) and \(\mathcal{P}\), interact with each other at point. (Edges are considered here as lines of intersection of the tooth flanks of a gear, \(\mathcal{G}\), and one of two gear faces.) The edge is formed by two surfaces: \(\mathcal{G}\) (or \(\mathcal{P}\)), and one of the gear/pinion faces. Under such an assumption, the edge can be considered as a line.

Lines of intersection of the tooth flanks, \(\mathcal{G}\) and \(\mathcal{P}\), by the plane of action, \(PA\), form an angle, \(\theta\).

In geometrically accurate gears, at every point \(K\) within the line of contact, \(LC\), the unit normal vectors, \(n_g\) and \(n_p\), to the interacting tooth flanks, \(\mathcal{G}\) and \(\mathcal{P}\), align to one another as depicted in Figure 5a. In this way, the unit normal vectors, \(n_g\) and \(n_p\), form an angle of 180°.

In misaligned gears, as shown in Figure 5b, the unit normal vectors, \(n_g\) and \(n_p\), form an angle that equals 180° - \(\theta\). This schematic (see Figure 5b) is a key to understand how a tolerance for the accuracy of the mounting distance can be calculated. No calculations of the tolerance for the accuracy of the mounting distance can be performed if a tolerance \(T(\theta)\) for the angle \(\theta\) is specified.

Even a small-axes misalignment in crossed-axes gearing (of about three angular minutes or so) results in a severe contact pattern change. The contact area should cover the entire flank surface (without edge contact concentration) if the nominal load rating is reached. An offset error of ±50 micrometers leads to a contact pattern change of about the same magnitude.

Under any and all circumstances, edge contact between the gear and pinion teeth has to be avoided. That is why, in order to keep the actual value of the angle \(\theta\) within a very tight tolerance for the accuracy of this parameter, tolerances for the accuracy of the design parameters and the axes misalignment depends on have to be set very tight.

As only the gear mesh is considered, just the unit normal vectors
n_g and n_p are taken into account, while the unit normal perpendiculars to the gear and the pinion faces are ignored.

No edge roundness, chamfers, etc. are involved in this analysis.

It should be mentioned that a proper shape along with a desired location and orientation of the contact pattern in a geometrically-accurate crossed-axes gear pair is the No. 1 priority from the standpoint of the actual value of the mounting distance.

4. ACCURACY OF THE MOUNTING DISTANCE IN CROSSED-AXES GEARING

In crossed-axes gearing, interaction of a gear and a mating pinion tooth flanks, \( \odot \) and \( \odot \), occurs within the plane of action, \( PA \). Therefore, it makes sense to consider the disposition of the base cone of a gear in relation to the plane of action. Later on, the results of the analysis obtained in this way can be applied to the disposition of the base cone of a mating pinion in relation to the gear housing.

4.1. DISPOSITION OF BASE CONE OF A GEAR IN RELATION OF THE PLANE OF ACTION.

In a geometrically accurate crossed-axes gear pair (that is, in a perfect crossed-axes gear pair) the axis of rotation of a gear, \( O_g \) and \( O_{pa} \), and of the plane of action, \( PA \), are crossed as illustrated in Figure 6a. The base cone apex of the gear, \( A_g \), and the plane-of-action apex, \( A_{pa} \), are at the center-distance, \( C \), from one another. A rotation of the gear is designated as \( \omega_g \), and a rotation of the plane of action is designated as \( \omega_{pa} \). Correspondingly. When the gears rotate, the base cone of the gear and the plane of action roll over each other only with axial sliding.

In reality, an error in mounting distance, \( \varepsilon_{pa} \), of the gear is always observed. The latter is shown in Figure 6b. In such a scenario, the gear base cone apex, \( A'_g \), does not coincide with the point, \( A_{hg} \), as \( A'_g \) is displaced axially in relation to \( A_{hg} \), at a distance \( \varepsilon_{g} \). In a case depicted in Figure 6b, the outward displacement, \( \varepsilon_{g} \), is of a positive value. An inward displacement is of a negative value (not shown in Figure 6b).

The performed analysis in Figure 6 allows the derivation of an equation for the calculation of the tolerance for the accuracy of the mounting distance in a crossed-axes gear pair.

4.2. DERIVATION OF EQUATION FOR THE CALCULATION OF TOLERANCE FOR THE ACCURACY OF THE MOUNTING DISTANCE.

Consider two teeth in contact intersected by the plane of action, \( PA \), as illustrated in Figure 7a. The teeth contact one another along a straight-line segment \( ab \). (Here, for simplicity, but without loss of generality, a pair of geometrically accurate straight bevel gears is considered.)

A case of outward displaced gear is illustrated in Figure 7b. As a result of the displacement, a gap between the teeth is observed. This gap is shown by two straight-line segments, \( ab \) and \( a'b' \), each of which is entirely located on the tooth flanks, \( \odot \) and \( \odot \), of the two interacting teeth.

A schematic for the derivation of a formula for the calculation of a tolerance for the accuracy of the mounting distance is depicted in Figure 7c. It should be stressed from the very beginning that point \( b \) within the straight-line segment \( ab \) is the closest point to the straight-line segment \( a'b' \). Therefore, an angle, \( \theta_g \), through which the plane of action, \( PA \), has to be turned about the axis of rotation, \( O_{pa} \), depends on the actual distance of point, \( b \), to the straight-line segment, \( a'b' \).

Consider the plane of action, \( PA \), and the angle, \( \theta_g \), of a gear and a mating pinion in a section by the plane of action, \( P_A \). Assume the gear is motionless, and the plane of action, \( PA \), turns about its axis of rotation, \( O_{pa} \), through an angle at which point, \( b \), touches the straight-line segment, \( A'_g \). In this triangle, \( A_{pa} A' bh \). Then, consider a triangle, \( A_{pa} A' b \). In this triangle, \( A_{pa} A' b = \sqrt{C^2 + \varepsilon_{g}^2} \). Then, either the actual value of the angle, \( \theta_g \), can be expressed in terms of the maximum permissible value of the angle \( \theta_g \), and the design parameters of the gear pair, or maximum permissible values of the displacement, \( \varepsilon_{g} \), can be expressed in terms of the maximum permissible value of the angle \( \theta_g \), and the design parameters of the gear pair:

\[
\theta_g = \cos^{-1} \left( \frac{r_{o, ap}^2 + (r_{o, pa})^2 - \varepsilon_{g}^2 - C^2}{2 \cdot r_{o, ap} \cdot \varepsilon_{g}} \right) \leq \theta_g
\]

Here, \( \varepsilon_{g} \) is the tolerance for the accuracy of the gear mounting distance, \( \varepsilon_{g} \):

\[
\varepsilon_{g} = \sqrt{r_{o, ap}^2 + (r_{o, pa})^2 - 2 \cdot r_{o, ap} \cdot \varepsilon_{g} \cdot \cos \theta_g - C^2} \leq \varepsilon_{g}
\]

Figure 7: Section of two interacting teeth by the plane of action, \( PA \): (a) zero mounting distance error, (b) mounting distance error of a certain value, \( \varepsilon_{g} \), and (c) schematic for derivation of the formula for the calculation of a tolerance for the accuracy of the mounting distance in crossed-axes gearing.

Figure 8: Possible kinds of modification of tooth flanks, \( \odot \) and \( \odot \), of a gear and a mating pinion in a section by the plane of action, \( P_A \): (a) straight gear-to-convex pinion, (b) convex gear-to-straight pinion, (c) convex gear-to-convex pinion, and (d) concave gear-to-convex pinion.
Here, $[\theta_g]$ is the tolerance for the accuracy of the gear angle, $\theta_g$. It can be shown (see Figure 7c) that the equality:

$$r^*_{o,pa} = \frac{r_{o,pa} \cos \varphi_{pa} - \mathbf{e}_g}{\cos \varphi_{pa}} \quad \text{Equation 4}$$

is valid.

When the pinion is fully aligned, the equalities $e = e_g$ and $\theta = \theta_g$ and Equation 2 and Equation 3 can be used for the calculation of tolerance on the mounting distance in the crossed-axes gear pair.

An analysis similar to that above, can be performed for a mating bevel pinion:

$$\mathcal{E}_p = \sqrt{\tau_{o,ap}^2 + (\mathcal{G}_p)^2} - 2r_{o,ap} \cdot \mathcal{E}_p \cos \theta_p - C^2 \leq [\mathcal{E}_p] \quad \text{Equation 5}$$

$$\theta_p = \cos^{-1} \left( \frac{2r_{o,ap} \cdot \mathcal{E}_p}{r_{o,ap} + \mathcal{G}_p^2 - \mathcal{E}_p^2 - C^2} \right) \leq [\theta_p] \quad \text{Equation 6}$$

Here, $[\mathcal{E}_p]$ is the tolerance for the accuracy of the pinion mounting distance, $\mathcal{E}_p$; and $[\theta_p]$ is the tolerance for the accuracy of the gear angle, $\theta_g$.

Further, when the gear is aligned, the equalities $e = e_g$ and $\theta = \theta_g$ and Equation 5 and Equation 6 can be used for the calculation of tolerance on the mounting distance in the crossed-axes gear pair.

Finally, in a more general case, both a bevel gear and a mating bevel pinion are misaligned. Under such a scenario, either the actual value of the angle, $\theta$, can be expressed in terms of the mounting distance errors, $e_g$ and $e_p$, and the design parameters of the gear pair, or a maximum permissible value of the displacements, $e_g$ and $e_p$, can be expressed in terms of the maximum permissible value of the angle $\theta$, and the design parameters of the gear pair.

The angle $\theta$ is formed by two perpendiculars, $\mathbf{n}_g$ and $\mathbf{n}_p$ [that is, $\theta = \angle(\mathbf{n}_g, \mathbf{n}_p)$], constructed at point of edge contact of a gear, $\mathcal{G}_p$ and a mating pinion, $\mathcal{G}_p$ tooth flanks, correspondingly: $\mathbf{n}_g$ is the unit normal vector to the gear tooth flank, $\mathcal{G}_p$ and $\mathbf{n}_p$ the unit normal vector to the pinion tooth flank, $\mathcal{G}_p$.

For the determination of the tolerances, $[e_g]$ and $[e_p]$, for the accuracy of the permissible axial displacements, $e_g$ and $e_p$, of the gear and the mating pinion, either one of the tolerances (either the tolerance $[e_g]$, or the tolerance $[e_p]$), or a ratio of the tolerances, $[e_g]/[e_p]$, has to be pre-specified.

The performed analysis reveals that the actual value of the angle, $\theta$, alters when the gears rotate. The maximum value of the angle, $\theta$, is observed at the very beginning of the meshing of two gear teeth. As the rotation proceeds, the angle, $\theta$, reduces to its minimum value. A minimum value of the angle, $\theta$, is observed within a plane through the axis of instant rotation, $P_{in}$, perpendicular to the plane of action, $\mathcal{G}_p$. Further, the angle, $\theta$, increases to its maximum value at the very end of meshing of two gear teeth.

A more detailed analysis is not presented here as the equations are bulky.

In addition to the discussed approach, another approach for the calculation of the tolerance for the accuracy of the mounting distance in crossed-axes gearing is developed.

When the displacements $e_g$ (or the displacement $e_p$) is of a negative value, this results in the edge contact occurring at the opposite face of the gear.

5. PERMISSIBLE ALTERATION TO BEVEL GEAR FLANK SURFACE GEOMETRY

As a gear and a mating pinion tooth interact with one another only within the plane of action, $\mathcal{G}_p$, the geometry of the tooth flanks, $\mathcal{G}_p$ and $\mathcal{G}_p$, allows for a slight modification (a few examples are illustrated in Figure 8) aiming avoiding edge contact in gearing. The modification is allowed only for lines of intersection of the tooth flanks, $\mathcal{G}_p$ and $\mathcal{G}_p$, by the plane of action. Only under such a scenario angular base pitch, $\varphi_{bg}$, of a gear equals to operating base pitch, $\varphi_{bop}$, of the gear pair (that is, the equality $\varphi_{bg} = \varphi_{bop}$ is valid); and similarly, angular base pitch, $\varphi_{bg}$ of a mating pinion equals to operating base pitch, $\varphi_{bop}$, of the gear pair (that is, the equality $\varphi_{bg} = \varphi_{bop}$ is valid).

No violation of the equalities $\varphi_{bg} = \varphi_{bop}$ and $\varphi_{bg} = \varphi_{bop}$ is permissible in precision crossed-axes gearing.

CONCLUSION

A favorable configuration of bevel gears in crossed-axes gearing is discussed in the article. It is stressed there that the mounting distance is one of the most important design parameters in crossed-axes gearing. The tolerance for the accuracy of the mounting distance has to be pretty tight and needs to be calculated. Calculation of the tolerance for the accuracy of the mounting distance in crossed-axes gearing is a challenging problem. No equations for such calculations are found in the public domain.

The disclosed approach for the calculation of tolerance for the accuracy of mounting distance in crossed-axes gearing is focused on geometrically accurate bevel gears (that is, on perfect bevel gears). However, it is also valid for the cases of approximate gears cut in a regular way on gear generators and so forth.

Bevel gears for crossed-axes gearing have to be designed (see [2]) to eliminate the necessity of “shimming-in/shimming-out” when assembling. These gears do not need in lapping when finishing the tooth flanks. The gears do not need to be paired, as they are interchangeable, and can be replaced individually (not as a whole gear set). If designed, machined [3], and assembled properly, no severe contact pattern changes are observed.

Understanding mounting distance is a key for the development of a method for the direct inspection of the accuracy of the mounting distance in crossed-axes gearing. Under an assumption, $C = 0$, the method is also applicable of the direct inspection of the accuracy of the mounting distance in intersected-axes gearing [1].

Accuracy of the mounting distance has to be inspected on the correct preload on the pinion shaft and gear carrier bearings. The gearbox housing accuracy and stiffness must be assured accordingly. Testing isn’t needed to verify the accuracy of the mounting distance if the gears and the housing are designed, machined, and assembled properly [2].

REFERENCES


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