PSYCHOACOUSTIC METHODOLOGY FOR THE NOISE REDUCTION OF BEVEL GEARS
How Fast Fourier Transformations of single flank errors can be used during the development of a gearset to establish harmonic levels and side bands of quiet rolling gearsets.

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The relationship between FFTs and gear quality as well as gear noise is investigated with some surprising conclusions, which leads to the discovery of a new interpretation of higher harmonics and the unknown influence of the residual error. An excursion into the psychoacoustic science in this phase of the research delivered many useful answers. Finally, a guideline was developed which proposes the combination of coordinate measurement, single flank test, structure borne noise, and additional analysis.

A rather exciting conclusion from the psychoacoustic research is the proposal of a gear transmission graph, which is a hybrid that connects different mathematical functions within the one pitch long contact area and outside of this area. A first surface development of a hypoid gearset has been realized by applying a UMC center section in connection with toe and heel sections, which are second order parabolas. The first results are very promising, which seem to confirm that the hybrid transmission function dramatically changes the way bevel and hypoid gearsets will be optimized in the future for silent operation.

INTRODUCTION

Noise in bevel gearsets has been a topic of research for many years. Single flank testing in connection with a Fast Fourier Transformation (FFT) has become a standard tool for development and production testing for the prediction of noise in the final application [6]. Inconsistent correlation between FFT results and the audible noise emissions from a bevel gearset can have many reasons. The discussion of a tooth contact analysis shows how the common characteristic of motion transmission functions is designed into the Ease-Off. The paper continues with explanations on several examples where it is not possible to capture non-harmonic and non-periodic motion error elements in the FFT. Also, these non-harmonic residual transmission elements will excite the gearset surrounding structures, while the structures will generate additional sound waves. The human ear recognizes mainly harmonic sound waves and expects them to be accompanied by integer multiples. The ear even complements missing frequencies within a spectrum of harmonic frequencies. These psychoacoustic phenomena are analyzed in the midsection of the paper. After a discussion about roll testing and the relationship between gear quality and operating noise, the section outlook introduces the conclusion of how an ideal transmission function can be derived. The conclusion bases on the gear meshing mechanics, the limitations of the Fourier analysis, and the psychoacoustic phenomena, which explains how the sound is received by a listener.

The main topics of this paper include:
- Designed flank surface crowning and theoretical motion transmission.
- Fourier analysis and its limits in gear noise analysis.
- Gear noise and psychoacoustics.
- Practical example of Fourier analysis and the residual phenomenon.
- Roll testing and relationship between gear quality and operating noise.
- Conclusion to introduce a sinusoidal hybrid motion transmission.

CONTACT ANALYSIS AND THEORETICAL MOTION TRANSMISSION ERROR

Traditional flank form optimizations in bevel and hypoid gears, and today also in cylindrical gears, use dominating second order modifications. A combination of circular length and profile crowning is shown in the Ease-Off in Figure 1. In bevel and hypoid gears, the crowning is partially applied to the pinion and partially to the gear. In cylindrical gears, it is common practice to manufacture one of the two members with-
out any modification and apply the entire crowning in the second member [4].

The top of Figure 1 shows the crowning amounts of two interacting pinion and gear flanks as topography plotted above surface, which represents the gear tooth boundaries, projected in an axial plane of the ring gear. The center graphic in Figure 1 shows the crowning along the path of contact. The bottom part of the graphic is a diagram of the gear angular deviation from a constant transmission with the correct ratio (ordinate) versus the pinion rotation angle (abscissa).

The Ease-Off in Figure 1 is cut in the path of contact direction with a plane, which traces the crowning along the path of contact. This path of contact crowning is shown in the mid-section of Figure 1. By dividing the ordinate values in direction S by the relevant radius of the gear, the bottom graphic in Figure 1, which represents the motion error, can be produced. The motion error graphic is only drawn from entrance to exit transfer point. Only this section is of interest for the following Fourier analysis because it is the area of transmission contact.

FOURIER ANALYSIS AND ITS LIMITS IN GEAR NOISE ANALYSIS

In order to approximate a complex graph, Brook Taylor introduced in 1715 the development of a series of polynomials. The series of polynomials have the form of:

\[ S(x) = \sum_{n=0}^{\infty} \frac{C_n}{n!} \cdot (x - x_0)^n \]  

where:

- \( S(x) \) ... Approximated function
- \( n = 1, 2, 3 \ldots \infty \)
- \( x_{\text{start}} \geq x \geq x_{\text{end}} \)
- \( x_0 \) ... Origin of polynomial
- \( C_n \) ... Coefficient order \( n \)

In the case that \( n \) becomes an infinite number, then the complex graph is precisely represented. If the task is to approximate a periodic graph, then a Taylor series is not suitable. Jean-Baptiste Joseph Fourier published in 1807 a development which uses a trigonometric series. Fourier's application was the solution of the equation for heat propagation in metal plates. There was no teaching by Fourier that his series development would capture and truly represent periodic events, such as the tone of an instrument in the summation of a certain number of orders. In the search for a suitable mathematical function, only trigonometric functions repeat themselves periodically. Among the trigonometric functions, only the sine-function is original, all other functions like cosine and tangents are derived from the sin-function. Fourier series development may not be ideal to capture certain dynamic and acoustic events, such as those commonly referred to as noise. It is based on the only analytical function available. Processing sinusoidal functions is significantly faster than processing “chained together” polynomials, wavelets, or even finite elements in order to represent dynamic events. Fourier analysis has been found to be very useful to analyze the elements of a dynamic event. The so-called harmonic amplitudes can be used in order to give a digital representation, for example, of gear noise. The Fourier series is written in the following form:

\[ S(x) = \frac{A_0}{2} + \sum_{n=1}^{N} A_n \cdot \sin(n \cdot \phi + \Delta \phi_n) \]  

where:

- \( S_N(x) \) ... Approximated harmonic function from 1st to \( N \)th harmonic
- \( n = 1, 2, 3 \ldots N \)
- \( x \ldots 0 \geq x \geq 1 \)
- \( N \rightarrow \infty \)
- \( A_0 \) ... Function offset
- \( A_n \) ... Amplitude of order \( n \)
- \( \phi \ldots 0 \geq \phi \geq 2\pi \)
- \( \Delta \phi_n \) ... phase shift of \( n \)th Order

Fourier analysis has an important place in gear noise evaluation in the form of the so-called “FFT” (Fast Fourier Transformation). A Fourier order analysis is the approximation of a harmonic signal as it is generated by a motion transmission error. In gearing, the order analysis is based on the tooth mesh frequency, also called the “fundamental frequency” or “the first harmonic frequency”:

\[ f_1 = \frac{(\text{Number of Pinion Teeth } \times \text{Pinion RPM})}{60} \]  

Equation 3

The multiples of \( f_1 \) are \( f_2 = 2 \cdot f_1 \), \( f_3 = 3 \cdot f_1 \), \( f_4 = 4 \cdot f_1 \) and so on. The amplitude levels at the different harmonic frequencies are the coefficients \( A_n (n = 1, 2, 3, 4 \text{ and so on}) \). If a gear engineer speaks about a first harmonic, then reference is made to the amplitude \( A_1 \) at the frequency \( f_1 \). In today’s noise evaluation practice, it appears to be an unquestioned fact that all noise characteristics of a steady state system are captured in the amplitudes of the Fourier analysis. However, there are three not-directly-obvious problems with this perception:

- Using only true sinusoidal functions.
- The residual, which cannot be captured by sine waves, is not recognized.
- Losing the time domain.

If the human ear only recognizes sine waves when it receives sound is a psychoacoustics question, which is discussed in the following section. The elements of the sound pressure waves that are not captured by the FFT are the residual errors. The residual errors make the difference in the “color” of a sound. The same applies to a motion transmission error. The residual signal will, in many cases, decide if a particular gearset is recognized when rolling on a test machine or in an automobile. The lost time domain means the relationship between the different orders (the phase shift \( \Delta \phi \)) is lost in the FFT results. The missing information can be acquired with a test series of different RPMs and viewed in a waterfall diagram or with a slowdown test cycle in a three-dimensional representation.

AN EXCURSION INTO PSYCHOACOUSTICS

Studying the vertical sequence of the graphics in Figure 2 makes it evident that according to the common acoustic interpretation, a square wave would cause a listener to hear several high frequencies. This raises a number of questions:

- Is the transmission of waves through the air or other media only possible in sine waves?
- If a true square wave was received by the human ear, would the brain first recognize the received wave form and replace it with a sine wave of the same frequency and similar intensity, and then substitute the higher frequencies, similar to Figure 2?
- If the human ear could hear the original square wave as a plain periodic plus-minus signal, would it sound the same as an artificial square wave that is a superimposition of four or more different frequency sine functions?

The questions above require some basic relationships between sound transmission and the psychoacoustics between the sound receiving by the human ear and its processing by the brain.
The airwaves actuate the eardrum, which in turn actuates the ossicles, compressor, which boosts the sound pressure by 15 to 20 dB. The conclusion is made that the ear, as a pneumatic-mechanical-hydraulic-electronic system, has masses, springs, and dampening and is created to recognize frequencies. It will mostly recognize harmonic air pressure changes, which are received in a periodic signal. However, an impulse will also be recognized, while ear and brain try to supplement the missing periodicity. The Fourier transformation of an impulse is shown in Figure 3. The impulse generates bars in the entire frequency range, which die down in amplitude as the frequency increases. This means that all those frequencies have been found by the Fourier transformation, although the impulse was an isolated occurrence with not really any frequency. The human ear will respond similarly because its design will cause an excitation of all hair cells along the windings of the cochlear and send signals of all audible frequencies to the brain.

Another interesting question is how a square wave sounds to the human ear compared to the results of a Fourier analysis. A perfect square wave, which is approximated with a Fourier series, shows an overshoot at the corners of the square (Gibbs phenomenon) [2, 3]. Figure 2 shows a square wave which is approximated by a first, third, fifth, and seventh order sine wave. The overshoot never dies out but approaches a finite limit of 9 percent of the square wave amplitude as the number of orders increases. The overshoots on the positive and negative half-wave represent together 18 percent of the amplitude, which is a significant amount, leading to a misrepresentation of this wave form. The question is, if the human ear, because of its function, will basically send a similar exaggerated signal to the brain when it receives a perfect square sound wave. This question might be academic because no sound generating source is capable of creating a perfectly square signal without distortion. The square wave has a ringing sound to the ear, which is attributed to the overshoot. Besides this, the airwaves would not be able to transmit such a signal without distortion. The square wave has a ringing sound to the ear, which is attributed to the overshoot. An additional peculiarity of the Fourier series of a square wave is that only the odd orders 1, 3, 5 ... are represented. The verification of the fundamental frequency of a square wave, which the ear conducts with the higher harmonics in numerous distinct areas of the tectorial membrane, is not given, and a strange hearing experience is the result. The square wave not only rings, it also sounds “cold and synthetic.”

Acoustical experiments with pure single frequency sine waves seem to confirm the theory that ear and brain will not complement the non-transmitted higher harmonic multiples of that sinusoidal sound. The pure single-frequency sine wave sounds smooth and rather quiet compared to a same-intensity square wave. This raises the question, will the ear, in the case of a parabolic motion error, notice the same higher harmonic frequency levels that result in a Fourier analysis of such a motion error? In many cases, the Fourier

Figure 2: Fourier series development for a square wave.

Figure 3: Impulse recognition.
analysis mirrors the psychoacoustics of the human ear very well (e.g. music) but also fails in many cases to deliver representative evaluation results (e.g. disturbing mechanical noise). Depending on the motion error characteristic, there are higher harmonics amplitudes in the FFT result and residual approximation errors, which are ignored. It is assumed that certain residual non-sinusoidal waves are audible, as distorted sine waves and certain harmonic amplitudes do not really exist in the sound waves received and processed by the ear. The non-existing harmonic amplitudes are merely a result of the Fourier summation scheme. It has been proposed to apply the smoother Fejér summation or Riesz summation or to use the continuous wavelet transformation in order to gain more relevant dynamic analysis results [9].

Dynamic analysis results have commonly two applications. One is the mentioned audible experience by humans, and the second is the conclusion to gear geometry-related manufacturing errors. The later asks for a sufficient qualitative and a concrete quantitative interpretation in order to allow for corrections in the machining process. The fundamental harmonics and the side bands can give certain hints to machining errors. The harmonics above the 4th order point, in some cases, at surface structural and roughness problems. Especially the second to fourth harmonic amplitudes can lead to the belief that there is, for example, a disturbance which occurs 2, 3, or 4 times at each tooth mesh. As a matter of fact, this is possible, but it might only be the result of the Fourier summation process required to capture a particular motion graph, which only repeats its disturbing rotational transmission once per tooth mesh.

EXAMPLE OF FOURIER ANALYSIS AND THE RESIDUAL PHENOMENON

In order to approximate a realistic motion transmission graph, a parabola of the form \( \Delta \phi = a \cdot (\phi - \phi_0)^2 \) as it is designed into a gearset (see Figure 1) is used in Figure 4 as the subject of a Fourier analysis. As a starting point, a sine-function with suitable amplitude and a period of the meshing time of one pitch is drawn into the parabola-shaped motion graph. If the first harmonic is subtracted from the motion transmission graph, the result is the first harmonic residual, which has twice the frequency of the motion transmission graph. This does not mean that the original motion transmission graph contained any elements of double frequency, it merely means that in the attempt to approximate a parabola with a sine-function, the residual will show a dominating second order. If the dynamic transmission media is capable of transmitting the original parabola-shaped sound wave, and if the receiver, e.g., a human ear, was rather capable in receiving and processing sinusoidal waves, only then would this second harmonic be noticed.

A sinusoidal function with half the period of the original function and an amplitude of about half of the residual magnitude is now used to approximate the residual function from the first harmonic. The residual from this approximation step result is shown in Figure 4 underneath the second harmonic. This graph appears to have some third and some fourth order elements. As a matter of fact, it requires the elimination of the third and fourth order harmonic elements in order to notice a visible reduction of the residual function. The unequal spacing of the waves makes it particularly difficult for a Fourier analysis to closely approximate a parabolic function. The residual error after the elimination of the third harmonic element still contains some first order residual, which can be determined at this point and then added to the amplitude \( A_1 \). The frequency-amplitude spectrums of the four sine waves in Figure 4 have been plotted into the graphic in Figure 5. Although this graphic shows the amplitudes for the dynamic gearset evaluation, only the first order peak-to-peak values are relevant, because only they represent the stroke of the excitation ripple and can be correlated to the transmission error.

There are several conclusions out of the experiment demonstrated with Figure 4:

- A true parabola-shaped graph was approximated. The result of a Fourier analysis of a perfect parabola-shaped motion transmission graph results in more than four harmonics of significant amplitudes. If the analysis is stopped after the four harmonic separation steps, a residual amplitude of about 50 percent of the original function is found.
- The value \( 2A_1 \) is larger than the original value of the motion error \( \Delta \phi_{\text{Gear}} \).
Many gear analyses are performed with only a four harmonics analysis. The residual function is so significant for the effective noise emission of the gearset that no absolute noise rating is possible.

FFTs have their value if they are used in the comparison of similar gearsets that have been found acceptable in their noise emission.

FFTs are useful if the presence of low frequencies caused by pinion and gear runout should be detected. Those waves are dominated by a sinusoidal content.

FFTs are useful if higher frequencies caused by surface texture or generating flats should be detected. Also, those waves are dominated by sinusoidal contents.

FFTs are useful in the medium frequency range (first to fourth harmonic) if the measured transmission variations have a dominating sinusoidal content.

It is impossible to capture a periodic impulse function with a harmonic Fourier analysis if the width of the impulse and the period of its reoccurrence have largely different amounts like shown on top of Figure 6. At the bottom, Figure 6 shows that the residual error reflects a high frequency with the amplitude of the original peak. In reality, the FFT will attempt to approximate the function and also interpret the impulse characteristic, which will result in side bands in the entire frequency range.

In another constructed example, the single flank error signal on top in Figure 7 consists only of sine functions. The top graphic is the recording of one ring gear revolution. In the example, a ratio of 3.00 was chosen, which means that the graphs in Figure 7 will be exactly repeated for additional ring gear revolutions. Figure 7 shows how in three steps, first the gear runout, then the pinion runout, and finally the tooth mesh is filtered out. In this example, no residual amplitudes are left. It can be assumed that a listener can clearly hear all three separated frequencies. At the bottom in Figure 7, the FFT result contains bars for the gear runout, the pinion runout, and the tooth mesh frequency. The side bands of the tooth mesh frequency originate from the gear and pinion runout. The side bands are spaced away from the tooth mesh frequency by their respective runout frequencies. Although the gear and pinion runout and even the generating flats commonly have a dominating sinusoidal shape, the tooth mesh is parabolic in most real cases, resulting in many additional frequency amplitudes, which is attributed to the transformation algorithm that is used in Fourier analysis and does not exactly represent the audible frequencies.

The development of the “Ultimate Motion Graph” in Figure 8 is targeted to noise reductions in ground bevel gearsets. Here for the first time, motion transmission graphs with non-parabolic shape are proposed [4]. The overall transmission error will not be the result of a single pair of teeth (like the green graph in the center) but will be the result of the interaction of three consecutive tooth pairs. At the entrance point, the measured motion error follows the green solid line from 1st to 2nd, and then the red solid line from 2nd to 3rd. After that, the motion error follows the green solid graph from 3rd to 4th, then the blue solid graph from 4th to 5th, and finally from 5th to the exit point, it follows the green solid line. The result is a graph with four unequally spaced waves, which shows lower amplitudes of motion error but also lower amplitudes in the FFT results. It should be noted that although a higher fourth harmonic FFT amplitude is expected compared to the parabolic graph, the FFT result of the Ultimate Motion Graph delivers a similar fourth order and a lower first order harmonic amplitude, but shows additional side bands between first and fourth harmonic amplitudes. Also, the Ultimate Motion Graph consists of parabolic elements and will cause certain residual amounts that are not captured and evaluated in the course of an FFT.

**ROLL TESTING AS FINAL QUALITY CHECK AND BUILT POSITION EVALUATION**

Motion transmission errors of real gearsets have their motion error not only from the design calculation, but in addition, from a variety of manufacturing related influences. Those imperfections come from runout in the workholding, generating flats and surface roughness, and also from first and higher order flank form errors.

Side bands are frequency amplitudes between the harmonic frequencies. They are generated by spacing variations or surface

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**Figure 5**: Frequency-amplitude spectrum of harmonic contents of a periodic parabola.

**Figure 6**: Residual error in the case of a periodic impulse.
structure shifts. Ghost harmonics are the amplitudes at frequencies that cannot be linked to gearset-related frequencies. Gearset-related frequencies are the rotational frequencies of pinion and gear, the tooth mesh frequency, the generating flat frequency, and frequencies additionally generated by the spacing variation or other periodic occurrences within a tooth mesh. Ghost harmonics are induced by the manufacturing machine, the surroundings of the machine, and previous operations, which might be a machining process or a metalurgic procedure. Typically, a peak, which is above the surrounding side band amplitudes, can be identified as a ghost harmonic.

The procedure of a single flank test is explained with the help of Figure 9 [6]. The gearset in Figure 9 is rotating in mesh with 30 to 60 pinion RPM where the pinion, for example, drives and the gear flanks are kept in contact with the pinion flanks by applying a light brake torque. Pinion and gear spindle are each connected with an encoder, which produces a sinusoidal signal. The angular difference between two encoder lines is now divided by the measured time durations \( T_1 \) and \( T_2 \) of one periodic signal, which results in the instant angular velocities \( \omega_1(t) \) and \( \omega_2(t) \) of the pinion and gear spindle. In order to gain the rotational angle \( \phi_1(t) \), the angular velocity samples of the pinion spindle are multiplied by the theoretical time duration between two encoder lines in seconds \( \Delta t_1 = \frac{60}{\text{RPM}_1 \cdot L_1} \) and summed up in real time with the sampling frequency. On the gear spindle, the same procedure is used. The difference signal can be obtained after the pinion \( \phi_2(t) \) is multiplied by \( z_1/z_2 \), because \( \Delta \phi \) is based on the gear rotation.

The knowledge about the tooth mesh frequency and the number of teeth is used to determine the section in the graph at the bottom of Figure 9, which represents one gear and one pinion revolution and a single tooth mesh. High precision encoders with 18,000 lines, for example, will also deliver information about tooth surface texture and roughness. If the point is made that the single flank test is a compound measurement that doesn’t show the errors or imperfections on an individual member, then this is correct, but it is also the strength of the single flank test. Only the errors relevant for the quiet meshing of the particular gearset are the result of a single flank test. If the waviness along the path of contact was measured with a tactile probe, then the results will not be applicable to an evaluation of the surface influence to the expected gearset noise if the mating member is not considered.

In Figure 10, the motion transmission error from Figure 9 is used as an example for the separation of the elements “gear runout,” “pinion runout,” and tooth mesh. The difference from Figure 7 is the tooth mesh motion error, which is parabolic in Figure 10 instead of sinusoidal. Due to the parabolic motion error, not only \( f_z \), but also the multiples of \( f_z \) are present in the Fourier analysis result. Each of those harmonic frequency bars is surrounded by side bands, caused by the gear and pinion runout.

ROLLED TESTING DURING DEVELOPMENT AND PRODUCTION

Roll testing is an end-of-production-line measurement of pinion and gear simultaneously. The knowledge about the number of teeth and the RPM enables the evaluation processor to separate pinion and gear runout as well as motion transmission error, and even surface texture.

During the gearset development, it is recommended to use the single flank test. The evaluation of the results should not be limited to the FFT amplitudes of the frequency spectrum, but should also consider the motion transmission error (see Figure 9), which is called “working variation,” in order to see the entire picture and not miss the 50 percent of lost information by the harmonic interpretation of a non-harmonic wave.
Transformation), will miss certain periodic, but not harmonic, contents of the test signal (residual errors). Both analysis methods are only justified by the worldwide established technique that gearsets which have been rated by a human test driver are related to a single flank test or a structure-borne noise analysis. If the FFT results include elements that reflect the test driver's subjective rating and the dB(A) evaluation of the microphone sound recording from the test drive, then the amplitudes of the tooth mesh harmonics as well as the side band structure of a gearset which was tested with a good result can be used to define the amplitude limits for the following production testing. This practice implies that the absolute amplitude values and frequency locations of the bars in an FFT plot are not the indicators of a quiet gearset, but that a quiet proven gearset allows the general conclusion that gearsets with the same specifications which show equal or lower amplitudes will also be quiet. It is unlikely, but possible, that certain acoustic elements that have not been captured by the FFT change their characteristic during the production run. Since those elements are not detected and rated, one of two gearsets with the identical FFT results could be rated “good” and the second could be rated “reject.”

SUMMARY
Fast Fourier transformations of single flank errors can be used during the development of a gearset to establish harmonic levels and side bands of quiet rolling gearsets. During the development, in addition to the FFT results, the measured single flank error (working variation) should be studied in order to gain an understanding of the non-sinusoidal transmission characteristics included in the graph, which lead to residual errors. Those elements are present in the single flank graph but cannot be captured with the FFT. It is also important to recognize that harmonics between the second and the sixth are generally not related to a dynamic problem with that frequency. Higher harmonics indicate, in most cases, that a summation of those orders was required to capture non-sinusoidal transmission characteristics. This means that with the fundamental frequency, a transmission characteristic is repeated that does not contain higher order frequencies, as the example of the square wave in Figure 2 showed. With the square wave, it was also demonstrated that the requirement for the FFT to employ higher orders was an indicator of a harsh and ringing sound. In comparison, a pure sine wave sounds smooth and is much less invasive.

The conclusion of this is that high importance should be paid to the Ease-Off and theoretical motion error development. Already during the theoretical development, FFTs of the motion error should be made and compared. The development target is not only the amplitude of the motion error and the size of the contact area, an additional target is the “design” of the FFT and the residual error. A software addition is proposed which can rate, during the theoretical development, the FFT results regarding the higher harmonics and the residuals and their implication to the first harmonic frequency noise. This software addition could show in the production rolling the magnitude and intensity of the residual error. This error is not recognized, but its change during a production batch indicates, for example, degradation in the manufacturing machine, the process conditions or the tool.

OUTLOOK
The following theses are the conclusion of this paper:
- Higher order harmonics found in an FFT are not really present as disturbances on the tooth surfaces.
- There is a considerable residual after an FFT of the motion error.
- The residuals might not be audible.
- The residuals indicate a non-captured dynamic disturbance.
- Changes in the residuals indicate part quality variations.
- The air transmits sound pressure in sinusoidal waves.
- The human ear, with its discrete frequency recognition of the tectorial membrane, mirrors the basic FFT function and only recognizes sinusoidal sound pressure waves.
The top graphic in Figure 11 shows the parabola-shaped transmission error graphs of three consecutive pair of teeth. Two adjacent parabolas always intersect at a point and continue below this point. In the case of no load, the motion transmission follows the red graph in the center drawing of Figure 11. In the case of the maximum load, the un-deformed transmission error in the lower graphic of Figure 11 changes to the deformed transmission error under load, drawn in blue. It can be noticed that the transmission error under load has a nearly sinusoidal characteristic. The results in Figure 11 have been generated using the Gleason Finite Element Software T900 [10].

An acoustic signal, consisting of a single fundamental sine function with a certain amplitude, sounds smooth and quiet, whereas an acoustic square wave signal with the same amplitude sounds harsh and loud.

This acknowledgement, in connection with the formulated theses, would allow the conclusion that a gearset with a true sinusoidal transmission error within the one pitch of single mesh (equal to the length of the tooth contact without load) like that shown on top in Figure 12 would sound extremely quiet under light or no load.

The sinusoidal transmission error on top in Figure 12 creates the conflict of the missing ability to adjust to increasing loads. Load increases above zero or light load (light load is less than 10 percent of the maximal load) would immediately cause edge contact, which in turn will make the gearset operation noisy with a high risk of tooth damages.

The solution proposed in this paper is the parabolic continuation below the intersecting points of the transmission graph, as shown in the center graphic in Figure 12. Below the intersecting points also means outside of the one-pitch-long active tooth contact. This hybrid between a sinusoidal and a parabolic transmission function will, under zero or light load, provide ideal sinusoidal excitation for a quiet gearset operation and will be equally suitable for all loads up to the maximal load the gearset is rated. The hybrid transmission error will change its shape under maximal load to a graph with reduced amplitude, which has still a dominating sinusoidal characteristic, as shown in the bottom graphic in Figure 12.

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