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By Ulrich Kissling and Markus Raabe

The variation of the tooth meshing stiffness is a primary source of noise. In production lines for gear manufacturing in large volumes, such as for the automobile industry, a gear is produced using many different tools with up to three pre-cutter processes and a finishing process (grinding, honing). Today calculation programs allow the entry of a tool and the input of a stock allowance for finishing. The simulation and exact calculation of the tooth form when using multiple tools is very important for the determination of the meshing stiffness, for the resistance calculation. For this purpose, the tooth calculation in the gear module of the KISSsoft software was rewritten with a completely new concept. An unlimited number of tools such as hob, cutting tool, gear-type cutter, grinding disk (generating or form grinding), and honing wheel can be defined in any sequence desired. The tooth shape after every step can be visualized, and different shapes can be superimposed to show the material removal from one step to another. The manufacturing process from tool to gear is also visualized, and sliding and rolling velocity vectors are indicated (to optimize the tool life).

The tooth form generated through exact simulation of the manufacturing process has many useful applications, such as tooth stiffness or resistance calculation. The variation of the tooth meshing stiffness during operation induces a deviation in the rotation-angle of the output gear from the nominal transmission ratio (transmission error), causing vibrations and noise. The meshing stiffness variation can be improved through optimization of the gear geometry (by optimizing the transverse contact ratio εα and the axial face contact ratio εβ), but the latest tendency is to apply a special wave form-like profile correction during the finishing process. For this purpose, the software will calculate the single tooth stiffness of any tooth form and determine the real contact path under load. From this information the real meshing stiffness of the gear pair is derived. It is possible to very quickly study the effect of a proposed profile correction on the behavior of a gear stage under different loads. The resistance calculation following AGMA2001 or ISO6336 checks the root stress in just one point. This procedure is therefore of limited value for special tooth profiles. If the tooth form is known exactly, the approach of the
AGMA method can be extended to every point of the tooth root and the point with the highest stress can be found. This method is very fast and gives good results (compared with FEM calculations). The effect of a grinding notch is also included in this procedure.

Tooth Form Calculation with Different Tools

In most of the available gear calculation software it is possible to calculate the tooth form when using a standard tool (hob, generating cutter, or gear-type cutter). Normally it is also possible to introduce a grinding allowance, and so simulate a two-step production process (cutting, then grinding).

Answering frequent requests of users of the widely recognized gear calculation software KISSsoft [3], it was decided to implement a new approach for tooth form calculation in the software. An unlimited number of tools such as hob, cutting tool, gear-type cutter, grinding disk (generating or form grinding), and honing wheel can be defined in any sequence desired. The tooth shape can be visualized after every step, and different shapes can be superimposed to show the material removal from one step to another. The manufacturing process from tool to gear is also visualized, and sliding and rolling velocity vectors are indicated (to optimize the tool life).

Fig. 1 shows the different stages in getting to the definitive tooth form when using two cutters and a final rectifying process. For the gear with a pressure angle $\alpha = 20^\circ$, a

**FIGURE 1: TOOTH FORM GENERATED WITH A THREE-TOOL PRODUCTION CYCLE:**

1) PRE-CUTTER $M_n=1.0368$, $\alpha_n = 25^\circ$ (GREEN);
2) PROTUBERANCE-CUTTER $M_n=1.0$, $\alpha_n = 20.0^\circ$ (BROWN);
3) FINAL RECTIFYING PROCESS (BLUE).
pre-cutter with an 25° is used. This leads to an increase of the tool service life and creates a better rounding of the tooth root. As the flank form should be identical to the final design, the base diameter \( db \) of the gear has to remain identical. Therefore the module of the tool must be increased by \( \cos(20°)/\cos(25°) \) for the pre-cutter. Some additional features of the tooth form calculation are:

- **Tool service life:** For the improvement of the tool service life (number of gears cut until the tool has to be resharpened or replaced) the display of the specific sliding on the tool cutting flank is very important (fig. 2). There are many factors influencing the tool service life. One of them is the local specific sliding on the tool. A high negative specific sliding on the tool implies that a short section on the cutting edge of the tool produces a large section on the gear. This means that this part of the tool is highly utilized, and consequently subject to high wear. As shown in fig.2, a pre-cutting-tool with higher pressure angle (as in fig.1) has a significantly reduced specific sliding.

- **Grinding notch:** For the root strength it is important to know if, through the grinding process, a so-called “grinding notch” (ISO6336-3, factor \( Y_{sg} \)) results. Therefore, during the calculation of the grinding process, the notch has to be recognized. Such a notch can reduce considerably the safety factor for bending stress. In KISSsoft [7] the corre-
Vibrations Caused by Tooth Mesh Stiffness Variation

The methods used in gear production are in constant development. In recent years form grinding (an alternative to the classic meshing grinding) has become the trend. By form grinding it is easy to apply a special wave form-like profile modification during the finishing process for the reduction of transmission error.

For many applications today the noise level is very critical, and it should be as low as possible. Noise is generated by transmission errors, which produce an incremental change of the velocity on the gear about the nominal value. This effect induces an instantaneous acceleration/deceleration into the transmission chain, and the result is vibrations. The transmission error is produced through the variation of the stiffness during a mesh cycle. It is also well known that fabrication errors generate a certain transmission error. Improvement of the gear quality helps to improve the situation, but even with a gear set of highest quality we have transmission errors due to the stiffness variation.

For this purpose the software calculates the single tooth stiffness of any tooth form and determines the real contact path under load. From this information the real transmission error of the gear pair is derived. It is possible to study very quickly the effect of a proposed profile modification on the behavior of a gear stage under different loads.

The calculation of the tooth pair stiffness under load permits a comparison of different gear geometries to find the best solution. Fig. 4 shows a typical case of a standard gear set (tooth form in fig. 3) with transverse contact ratio ($\epsilon_\alpha$) = 1.67 ($z$: 25:76, DP 4.2333, module 6.0). The transmission error is low (stiffness is high) during 67 percent of the cycle, when two pairs of teeth are in contact, and otherwise high when only one tooth pair is in contact. (Remark: A theoretical transverse value of 1.67 signifies that during 67 percent of the time two pairs of teeth are in contact, and during 33 percent only one pair.) The gear without profile modification experiences a sudden “jump”
from low to high transmission deviation, as shown in fig. 4. A well designed profile modification would reduce the rate (or steepness) of this jump, producing a smoother change in the stiffness, but would not change the level of the low and high stiffness phase. So profile modification is useful, but not the solution to the problem. The best method to reduce the variation of the stiffness is to use a deep tooth profile with, theoretically, a transverse contact ratio of 2.0. In this case, as two pairs of teeth are always in contact, the jump is eliminated.

Best results will be achieved when the third tooth coming into contact is unloaded. This is possible when an appropriate profile modification is applied. In this case the theoretical contact ratio has to be higher than 2.0, according to the length of profile correction planned.
The Calculation of the Transmission Error Due to Tooth Mesh Stiffness

The stiffness of a single tooth is composed of four important effects. These are:

- Tooth bending
- Shear deformation of the tooth
- Hertzian compression in the contact
- Tilting of the tooth in the gear body

Equations for the calculation of these effects were developed by Peterson [4] for involute gears. The method can also be adapted to gears with profile modification and to non-involute gears. The stiffness of two teeth in contact is called “combined tooth stiffness of one pair of teeth” \( c' \), and the total stiffness of all teeth in contact is the “mesh stiffness” \( c_y \).

The calculation of the behavior of the meshing stiffness during a contact cycle is important. For unloaded involute gears with no profile modification the path of contact is a straight line, but for real gears under load the effective path of contact is complicated to find. Due to bending of the teeth, the real transverse contact ratio \( \varepsilon \alpha \) increases. Therefore, the contact path has to be calculated step by step; and for every single step, the effective number of teeth in contact has to be determined. As the tooth stiffness depends on the load, and the load on a single tooth (with constant torque on the pinion) depends on the number of teeth in contact, the solution must be found through iteration (Fig. 6). The calculation is time consuming because the stiffness itself depends on the applied normal force (due to the Hertzian compression); therefore every single step on the contact path has to be found by double iteration.

The result is quite impressive. Fig 4 shows a typical example of a gear set without (top) and with different degrees of profile modification (middle and bottom). The technique of displaying transmission error under varying load such as 25, 50, 75 and 100 percent of the nominal torque load, and defined as far back as 1958 by Harris [5] is a very helpful illustration for evaluating the behavior of a proposed profile modification. Most gearboxes do not always run with a steady torque, so the performance of the gear set should be optimal within a cer-
tain torque range. The transmission error of the unmodified gear in fig. 4 also shows an often-discussed feature of gears: due to the flexure of the teeth, the transverse contact ratio (εα) increases with higher torque from 1.67 to approximately 1.78 (50 percent load) and 1.81 (at 100 percent load).

Any small modification on the tooth profile has an important influence on the transmission error curve, as the comparison of the different gear sets in fig. 4 shows. A new tendency in the development of optimized gears is to apply a special profile modification during the finishing process for the reduction of transmission error. When increasing the thickness of the tooth between the limits of the single contact diameters (the section of the tooth flank with one tooth pair in contact), the value of the transmission error in this section can be reduced. To demonstrate this, fig. 5 presents the transmission error showing the effect of a sinus wave-like modification. It is evident that such a modification produces higher transmission errors in low load conditions, but in the nominal torque range the change of the transmission error is...
significantly reduced. The behavior of the transmission error can be further improved when using other modification forms. The sinus wave was chosen arbitrarily to demonstrate the principle.

It is well known that fabrication errors (such as pitch deviation) have an important influence on the transmission error of real gears. These effects can be analyzed by applying an error to the tooth form. For example, the distance between two flanks can be increased by the amount of the pitch deviation error before calculating the transmission error.

Gear Stress Analysis for Optimized Involute Tooth Forms

The current calculation method for the tooth resistance following either AGMA2001 [1] or ISO6336 [2] is based on the assumption of a tooth form produced by one tool in a meshing process. When using a tool with protuberance, the method also includes a production process with a pre-cutter (with stock allowance for finishing) and a final grinding or honing process.

This implies that the formulas in AGMA or ISO resistance calculation methods cannot be applied with gears produced by form grinding or other non-conventional methods. The problem is that, for the calculation of the tooth root stresses, some values such as tooth thickness and root rounding must be known. The calculation method assumes that the tooth form is not exactly known, and therefore presents formulas which permit calculation of the tooth form just in the considered section of the tooth. These formulas assume production through a meshing process. But, in principle, if the tooth form is given, the tooth can be calculated by directly using the formulas proposed by the standards. Therefore, if the tooth form calculation is integrated into the resistance calculation software, AGMA or ISO standards can be used for any production method.

A reliable algorithm was developed for the strength calculation of optimized gears. All normal calculation procedures determine the stress in the tooth root via a simplified model of the real conditions. According to ISO 6336, the critical cross section in the tooth root has to be found at the contact point of the 30° tangent in the root contour. According to AGMA2001 (with AGMA908-B89), the Lewis parabola
is fitted to the tooth form, where the point of contact of the parabola with the tooth root rounding determines the critical cross section. Depending on the actual shape of the tooth root rounding, a more or less greater error is implied. In the publication of B. Obsieger [6] some years ago, a substantially improved calculation method was proposed. Based on the actual tooth form, the tooth form factor (ISO:YF, AGMA:Y) and the stress correction factor (ISO:YS, AGMA:Kf) are calculated at each point in the tooth root area and, subsequently, the location of the maximum of the product (ISO:YF*YS, AGMA:Y/Kf) is determined (fig. 7).

This calculation procedure is integrated into the KISSsoft
software. The critical tooth root cross section can be determined based either on the force application on the tip (AGMA: loaded at tip) or the force application at the single point of action (AGMA: loaded at PSTC). The strength calculation according to ISO or AGMA is then completely performed using this

FIGURE 8: TOOTH ROOT STRESSES (AND HERTZIAN PRESSURE)
calculated on the base of the actual tooth form and the meshing conditions.

FIGURE 9: LEFT: NORMAL FORCE.
RIGHT: HERTZIAN STRESS AND TOOTH ROOT STRESS DURING A MESHING CYCLE. (SAME GEARS AS IN FIG. 4 AT 100% LOAD.)
specific data. The course of the stresses can also be graphically shown (fig. 8). This method gives good results when compared with FEM calculations. The effect of a grinding notch is also included in this procedure. For a resistance verification following either AGMA or ISO, the point of maximum stress is identified, and the calculation is executed at this specific point. As the ISO standard explicitly stipulates a method A, for an improved algorithm based on the same basic philosophy, this type of calculation is 100 percent in accordance.

The computation of the Hertzian pressure along the tooth flank is also calculated based on the actual tooth form. For each point of action the corresponding radii of curvature are determined for both gears, and starting from this the pressure is computed.

With the same data the calculation of the sliding speed is possible, as well as the computation of the local contact temperature, the efficiency, and the safety factor for scoring of any gear pair.

**Gear Stress Analysis Over the Path of Contact**

The procedure for the calculation of the transmission error (fig. 6) also provides the normal force and the point of contact on the tooth during the meshing cycle. This permits an even more precise analysis. Fig. 8 shows the Hertzian pressure and the maximal tooth stress calculated with the actual normal force.

As the standards (AGMA, ISO) are based on the calculation of just one situation—i.e. the application of the normal force in the point of single tooth contact—the more realistic calculation of the normal force with the corresponding stresses in every position is far

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more precise. However, the result may be quite different from the results of the procedure in the standards.

Conclusion

Using advanced software for the simulation of the manufacturing process with one or more different tools and production methods, the effective tooth form is calculated. Based on this data, the calculation of the root resistance and transmission error under load is performed. An enlargement of the root resistance calculation is proposed, so either by AGMA or ISO standards can be applied to any tooth form; form-ground gears, for example. For the development of low-noise gear sets, the calculation of the transmission error under different loads is very important. The software permits the study of the effects of different profile modifications on the behavior of the gear set under load, and to find the best solution for the required torque range.

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