Striving for High Load Capacity and Low Noise Excitation in Gear Design

The conflict between flank modifications, with a focus on load capacity, is documented.

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MANY DIMENSIONS OF THE GEARS AND THE TOOTHING ARE DETERMINED THAT HAVE SIGNIFICANT INFLUENCE ON LOAD CAPACITY AND NOISE EXCITATION. THOUGH A DETAILED ANALYSIS IS NOT IN FOCUS YET, THE FINAL GOAL OF OPTIMIZATION HAS TO BE KEPT IN MIND.

The last step of optimization usually is an adequate design of flank modifications. The possibility to improve the gear mesh behavior by flank modification is closely dependent on the selected main geometry of the toothing.

A different approach, e.g., starting with a detailed FE-Simulation of the whole gearbox, is not efficient. Complexity of the model and number of necessary parameters are so big so that it severely limits the flexibility that is needed in the early stage of the design process. A fast approach to flank microgeometry for low noise excitation is described by Houser and Harianto [9].

A variation of microgeometries is analyzed by an analytical method and an interactive selection allows the identification of a desired result. The modifications are designed to also account for the influence of manufacturing deviations. For increasing normal module, that influence gets less significant, since other parameters (clearances, deformations) amount to most of the resulting deviations in contact.

A method with focus on load distribution is discussed in [7] by Thoma et al., the tooth contact analysis includes the influences of bearings and shafts. The load distribution resulting from the calculation is used to evaluate load distribution factors according to ISO 6336-1.

Pears et al. [15] describes an evaluation method for the contact stiffness in the mesh that is used for an analysis of transmission error. The influences of further elements like shafts and bearings are included. The process described in this paper starts with rough design methods and successively proceeds to more detailed methods. Necessary calculations are carried out by several specialized software programs. The programs from early design stages provide data for latter, more complex steps and programs. By this approach flexibility stays high and necessary design changes can be realized even late in the design process. The resulting data are passed on to the more complex analyses without obstacles between the different programs. Data management is controlled by a unified User Interface for all programs (FVA Workbench [4], see Figure 1).

Succession of design steps:

- Define gearbox structure;
- Distribute the ratios of the different stages to reach the gearbox ratio (GAP [1]);
- Design main geometry of the gears (STplus [8]) according to load capacity requirements;
- Dimension shafts and bearings, perform detailed calculation of shaft deformation and bearing lifetime
- (RIKOR/WELLAG [16],[18]), include housing elasticity and further influences;
- Determine load distribution by tooth contact analysis (RIKOR [16]);
- Design flank modifications to reach high load capacity, check for noise excitation;
- Optimize flank modifications for low noise excitation (DZP [5]).

Finally, the goals high load capacity and low noise excitations are reached by designing flank modifications. The goal high load capacity leads to a detailed definition of the flank modifications to ensure even load distribution, usually at high loads. Noise excitation behavior

Figure 1: Interaction of programs in design process.

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may require a different microgeometry, because the noise relevant operating range is related to lower torque moments. A tradeoff between the two goals is required to determine the final flank modifications.

GEAR MAIN GEOMETRY

As an example, a gearbox structure with two helical gear stages and one planetary gear stage will be used (see Figure 2). The focus in this paper will be the high speed stage. For this given structure, the ratios of the stages have to be determined. The goal here is to reach a design that is compact and light. The ratio is an important factor for the mass of the gears. The torque moment transmitted in the mesh is the main influence for the dimension of the teeth. With given power, low speed shafts transmit a higher torque moment than high speed shafts. This leads to bigger teeth and wider gears on low speed shafts and ultimately a higher mass of the respective stages. Therefore a planetary stage with the advantage of internal power split is used for the low speed stage and helical stages are selected for the higher speeds.

Determining the ratios of the stages is an optimizing task that is performed by the program GAP according to suggestions by Winter [14] or Linke [13].

It is necessary to rate the load capacity of the gear meshes in the optimization process. This can be done by simple methods like $K^*$ value for flank load capacity and $U$ value for tooth root capacity.

\[
K^* = \frac{F_t}{b d_1} \frac{u + 1}{u}
\]

(1)

\[
U = \frac{F_t}{b m_t}
\]

(2)

$K^*$ is flank load capacity;
$F_t$ is load in transverse plane, N;
b is face width, mm;
d_1$ is reference diameter, mm;
$u$ is gear ratio;
$U$ is tooth root capacity;
m_t is transverse plane module.

Since the final geometry has to be rated according to ISO 6336 [10], ISO 6336 is used in the basic step as well. For this purpose, many default values are assumed, e.g., for the $K$-factors of ISO 6336, tool data and further values. This has the advantage that the default values can be determined in more detail during the design process without leaving the framework of ISO 6336.

With the ratios defined, the torque moment and speed of every stage is determined. Table 1 shows the main geometry of the high speed shaft. The detailed tooth geometry can be defined including manufacturing and heat treatment. A geometry calculation with respect to cutting and grinding tool geometry is necessary. The final geometry includes backlash and detailed tooth root contour. On that basis, a full load capacity calculation according to ISO 6336 is performed. This is easily accomplished with the STplus-suite [8] that includes the geometry calculation and the standard calculation as well.

To cover pitting resistance and risk of tooth root breakage, ISO 6336 is adequate. Further limiting factors can be taken into account e.g., by ISO TR 13989 (scuffing) [11], ISO TR 15144 (micropitting) [12] and further standards like API or ABS. Being available in direct connection to the geometry calculation makes the application of these additional methods easy and the process straightforward. Figure 3 shows the tooth form in the mesh and the range of standards that are available for rating the gears in the mesh.

SHAFTS AND BEARINGS

Dimensions of shafts and bearings are covered only by rough estimate at first. A detailed calculation has to follow to ensure sufficient load capacity. To determine the influence on the contact conditions in the mesh, the deformation of the shaft-bearing system has to be calculated.

The shaft deformation is well handled by using linear beam theory. The common approach neglects the shear forces on the deformation which is reasonable for slender shafts. Gearbox shafts have a length to diameter
To cover roller bearings sufficiently, bearing clearance and the contact deformations between rollers and raceways have to be included (Eschmann [3]).

In roller bearings the load is distributed between the rollers that transmit the contact loads. Dependent on the type of bearing, the analysis may have to cover up to five degrees of freedom (three linear axes, two rotational axes). A taper roller bearing, that is widely used for high load capacity, transmits not only radial and axial forces but reacts with a bending moment on shaft inclinations. A result in this detail is computed by covering each contact between roller and raceway. The line loads in the contacts are determined in respect to the position of raceway, roller and profile of the contact partners. When not all of these detailed information is available, reasonable assumptions are made (e.g., logarithmic profiles of rollers). With calculation methods for shaft and bearing (WELLAG/LAGER2 [16]) the elements can be considered in detail. On the basis of these results, contact pressures in the bearing can be calculated and an advanced bearing lifetime calculation according to DIN 26218 (DIN ISO 281 add. 4) [2] is performed.

The shaft deformation and the bearing deformation are closely coupled and have to be determined in respect to each other (Figure 4).

Figure 5: Example roller bearing, the left part shows the contact pressure for every roller with inner raceway. The roller is divided in slices, width of slices decreases at the edges to ensure accurate results (RIKOR/LAGER2) [16] [18].

Table 1: Main gear data.

<table>
<thead>
<tr>
<th>High speed stage</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth, (z)</td>
<td>35</td>
<td>138</td>
</tr>
<tr>
<td>Gear ratio, (u)</td>
<td>3.943</td>
<td></td>
</tr>
<tr>
<td>Normal module, (m_x), mm</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>Center distance, (a), mm</td>
<td>695.0</td>
<td></td>
</tr>
<tr>
<td>Normal pressure angle, (\alpha_n), degrees</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>Helix angle, (\beta), degrees</td>
<td>13.0</td>
<td></td>
</tr>
<tr>
<td>Profile shift coefficient, (x)</td>
<td>0.4020</td>
<td>0.2267</td>
</tr>
<tr>
<td>Tip diameter, (d_t), mm</td>
<td>292.399</td>
<td>1087.518</td>
</tr>
<tr>
<td>Face width, (b), mm</td>
<td>235.0</td>
<td>220.0</td>
</tr>
<tr>
<td>Transverse contact ratio, (e_t)</td>
<td>1.833</td>
<td></td>
</tr>
<tr>
<td>Overlap ratio, (e_o)</td>
<td>2.005</td>
<td></td>
</tr>
<tr>
<td>Total contact ratio, (e_c)</td>
<td>3.838</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4: Effects of shaft deformations and of bearing clearance ratio that may be below two and cannot be considered as slender. The equations are used including the effects of shear stress [16], [17].

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To account for the close coupling, equilibrium between shaft and bearing forces has to be achieved. This is performed by an iteration, because the function of inner ring dislocation in respect to the outer ring of the bearing may be highly non-linear (however, more linear behavior is supposed for taper roller bearings under axial preload), see Figure 5. A valid result has been found when forces and deflections of shaft and all bearings on the shaft are compatible.

Important influences on the position of the shafts and on the contact conditions in the gear meshes may result from housing deformations. Since the housing is of complex geometry, a FE approach is encouraged. The resulting deflections of the bearing positions in the housing are transferred to the gear calculation.

The results of these efforts not only yield the load capacity of shaft and bearings and the deformation, that has to be considered in the mesh, but also the robustness of the design. To have a good basis for microgeometry design, the contact condition should be stable over a wide load range. This is strongly influenced by the design of the shaft and bearing system, even the housing may have to be considered as mentioned above.

A design value for the overlap ratio is only valid if the tooth flank is engaged over the whole width and the overlap is in working contact. For the example gearbox shown in Figure 2, the design becomes more robust if the shaft geometry and the bearing location are changed, see Figure 6. In the design with the longer shaft, the intermediate stage and the final stage have swapped positions. The pinion is moved closer to the supporting bearing on the right. Shaft deformation then leads to an inclination of the gear and to deviations in the mesh. However, the bending deformation now compensates the torsional deformation of the pinion that is also acting in the mesh. Since both effects are proportional to the applied load, contact conditions in the mesh are much more stable with the changed design over a wide load range.

**TOOTH CONTACT ANALYSIS**

Contact pattern, load and pressure distribution and further values can be derived from the tooth contact analysis. The displacement and the elasticity resulting from the shaft-bearing system and the housing are transformed into the plane of action and included in a system of equations. Important input data are the elasticity matrices of the teeth and the flank microgeometry. These data allow an analysis of the load distribution along the contact lines. To evaluate the loads acting on the flank during the whole mesh, several different meshing position have to be analyzed [16].

The meshing stiffness is varying over the path of contact and results from the superposition of the tooth pair stiffnesses, see Figure 7. For each meshing position, the influence of the environment (shafts, bearings, housing) is superposed.

Solution of the system of equations is the load distribution in the mesh. Further data can be acquired on that basis: tooth root stress, film thickness or flash temperature. These values are indicators for different limits of capacity.

\[
\left( \frac{F_b}{b} \right)_{3D, \xi} = \frac{\varepsilon c}{n} \sum_{i=1}^{n} f_b, 3D, \xi, i
\]

where

- \( F_b \) is load face width, mm;
- \( D \) is diameter, mm;
- \( n \) is rotational speed, min\(^{-1}\);
- \( f_b \) is face width deformation, mm;

**GEAR MICRO GEOMETRY**

**FOCUS LOAD CARRYING CAPACITY**

Basic assumption in the design of microgeometry for high load capacity is that an even load distribution in each meshing position complies with uniform utilization of the material and leads to an optimal load carrying capacity. This goal is reached by solving the system of equations...
with an even load distribution as input and the microgeometry as unknown vector. The result satisfies the assumptions, but is only optimal for the given load. To compensate for manufacturing deviations, varying loads and other uncertainties, a microgeometry with a certain amount of crowning or end relief hast to be designed. This may lead to partial contact pattern under partial load and may have a negative effect on noise excitation.

In Figure 8, a pressure distribution is shown that results in low load at the corners of the active flank. For the load that is relevant for load capacity calculation, this provides an advantageous load distribution. Under partial load, the contact pattern doesn’t cover the whole flank any more. Especially for the partial load that is important for noise excitation this has to be inspected closely.

**FOCUS NOISE EXCITATION**

The main source of noise excitation in a gearbox is the gear mesh. The varying meshing stiffness leads to transmission error. In extension to the approach for load capacity that focuses on the load distribution in each meshing position separately, the focus for noise excitation is the deformation in successive meshing position. This is dependent on the stiffness variation over the mesh.

When designing a flank modification to achieve low noise excitation, the goal is not to provide uniform load distribution in the first place but to compensate for the changing mesh stiffness. Similar to the flank modifications derived from capacity considerations, modifications for low noise excitation have one optimum design load.

In the example gearbox, the gear main geometry was designed according to common rules for low noise excitation. Applying the modifications for sufficient load capacity, the tooth contact area and load distribution in the mesh are not acting on the whole flank for partial load. The basic design assumptions are not fully valid any more.

To rate the noise excitation behavior for different modifications in comparison to each other, the tooth force level is supposed. Figure 9 shows that the flank modifications designed for high load capacity do not lead to a lower noise excitation level than the unmodified gear mesh. Since sufficient load capacity has to be granted, the parameters available for variation and optimization of the flank modifications are very limited.

**OPTIMIZATION**

Useful for influencing the noise excitation behavior are modifications that have significant influence along the path of contact that means in the succession of meshing positions. Instead of a manual try and error procedure an automatic approach is used. Certain forms of profile and flank modifications are selected and a variation program combines the modification forms and calculates the tooth force level. The result is compared with the last results and dependent on a possible improvement, the last flank modifications are used as new reference for the next variation.

The aspects of high load carrying capacity and low noise excitation shall be connected. Since each aspect alone leads to a tight definition of the modifications, a compromise has to be
Different modifications (tip/root relief, flank twist, crowning) are applied in a simulated annealing algorithm. As additional constraint the maximal pressure in the pressure distribution is limited. To provide some maneuvering space for the algorithm to reach a compromise, the maximal pressure is limited to 10% above the optimal value that has been achieved with the optimal flank modifications for load capacity. Under this constraint the algorithm searches for the lowest noise excitation.

Figure 10 shows the tooth force level of the resulting modification in comparison to the unmodified flank and the load capacity modification. The typical behavior for modified gears with main geometry design for low noise excitation is visible: An optimum is reached at a design load. At lower loads a higher noise excitation level than the unmodified gear is calculated. At higher loads noise excitation level remains below the gear with modifications for load capacity.

In Figure 11, the flank modifications for load capacity and for low noise excitation are shown. The differences are quite small, so much finer determination of the modification parameters is necessary in order to minimize noise excitation in comparison to achieve a good load distribution.

**DYNAMIC ANALYSIS**
Defining microgeometry for low noise excitation is one part, the other important part is checking any eigenfrequencies that are excited by the gear mesh. Meshing frequency and its harmonics are determined by the number of teeth and the rotational speed. The first approach that should be used already at an early design stage is a simple one. The formulas can be taken from ISO 6336-1.

\[
N = \frac{n_t}{n_E} = \frac{n_t z_1}{30000 \sqrt{m_{red} c_{\gamma \alpha}}}
\]

Where
- \(n_t\) is resonance rotational speed, min\(^{-1}\);
- \(m_{red}\) is reduced mass, kg;
- \(c_{\gamma \alpha}\) is tooth pair stiffness, N/mm

The method covers rotational oscillation of the masses of the gear bodies that are connected by the meshing stiffness against each other. This eigenfrequency of the gear mesh has to be avoided during use. Not only leads operation in this range to high noise excitation but also to high dynamic loads, so that load capacity can be critical. ISO 6336-1 suggests to avoid operation in a range of 0.85 < N < 1.15.

Using the simple approach, only that single eigenfrequency is determined, since only one mass is included. To get a more detailed picture of the behavior of the gearbox, multi body simulations can be used. Being most precise requires a time step integration to account for the changing mesh stiffness. This is an elaborate task that is time consuming and may not be a process that is done early in the design process.

An approach between these degrees of complexity is provided by the method indicated in Figure 12, a detailed multi-body-system of the shafts, bearings and gear meshes is built up. Instead of time step integration considering the real mesh stiffnesses, a median value is used for each mesh, similar to \(c_{\gamma \alpha}\) from ISO 6336-1. To include the excitation from the mesh, the transmission error (as equivalent tooth force excitation) is applied to the model. Then a fast solution in frequency domain is possible. With these results a model reduction algorithm is applied to the model. Then a fast solution in frequency domain is possible.

Figure 11: Resulting flank modifications in the mesh for load capacity (left) and for low noise excitation (right).
CONCLUSION

The goals load capacity and noise excitation meet at the design of flank modifications. The conflict between flank modifications with focus on load capacity and with focus on noise excitation is documented. To reach an acceptable compromise, an optimization task is formulated for the resulting flank modifications.

Besides the steps of tooth contact analysis the eigenfrequencies of the system have to be respected. A simple approach is documented, a more thorough method is named but not discussed in detail. The design tasks have been covered by various computer programs from FZG that are interconnected by FVA Workbench. Today's complex program systems based on FE-methods or MBS in many cases are reasonable tools to evaluate the final design.

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