In this article you’ll learn about a fillet profile optimization technique that allows for substantial bending stress reduction, producing a variety of gear performance benefits.

By Alex Kapelevich and Yuriy Shekhtman
Historically, gear geometry improvement efforts were concentrated on the working involute flanks. They are nominally well described and classified by different standard accuracy grades, depending on gear application and defining their tolerance limits for such parameters as runout, profile, lead, pitch variation, and others. Working involute flanks are also modified to localize a bearing contact and provide required performance at different tolerance combinations and possible misalignment as a result of operating conditions (temperature, loads, etc.). Their accuracy is thoroughly controlled by gear inspection machines.

The gear tooth fillet is an area of maximum bending stress concentration. However, its profile and accuracy are marginally defined on the gear drawing by typically very generous root diameter tolerance and, in some cases, by the minimum fillet radius, which is difficult to inspect. In fact, tooth bending strength improvement is usually provided by gear technology—case hardening and shot peening to create compressive residual stress layer, for example—rather than gear geometry.

The gear tooth fillet profile is typically the generating cutting tool (gear hob or shaper cutter) tooth tip trajectory (fig. 1), also called trochoid. If the cutter parameters are chosen or designed to generate the involute flank profile, which must work for the certain gear application and satisfy certain operation conditions, the fillet profile is just a byproduct of the

Fig. 1: Gear tooth fillet generation by the rack cutter (gear hob); (1) cutter tooth tip; (2) gear tooth fillet, as a trajectory of the cutter tooth tip; (α) rack profile (pressure) angle; (A) addendum; (C) radial clearance; (R) cutter tip radius.
cutter motion. The fillet profile and, as a result, bending stress are also depended on the cutter radial clearance and tip radius. The standard radial clearances usually are $0.25/P$ or $0.20/P+0.002”$, where $P$ is the standard diametral pitch. The standard cutter tooth radius for the coarse pitch gears is $0.3/P$. For the fine pitch gears the standard cutter tooth radius is not standardized and can be as low as zero [1].

Unlike the contact Hertz stress, the bending stress does not define the major dimensions or the gears, such as pitch diameters or center distance. If the calculated bending stress is too high, in many cases the number or teeth can be reduced and the coarser diametral pitch (larger module) can be applied to keep the same pitch diameters, center distance, and the same (or close) gear ratio. This makes gear tooth physically larger and reduces bending stress to the acceptable level. Of course, this increases specific sliding and reduces contact ratio and gear mesh efficiency, but this is better than the broken teeth.

There are two general approaches to reduce bending stress for the given tooth size. One of them is to alter the generating cutter tooth tip, and the most common application of this approach is the rack with the
full tip radius. Another approach is to alter the gear tooth fillet profile and the most common application here is the circular (instead trochoid) fillet. Further development of both these approaches is based on a mathematical function fitting technique when the cutter tip radius or the gear tooth trochoid fillet profile is replaced by a parabola, ellipse, chain curve, or other curve reducing the bending stress [2, 3]. Bending stress reduction achieved by such fillet profile improvement differs and greatly depends on the cutter or gear tooth parameters. The resulting tooth fillet profile must be checked on interference with mating gear at various gear (and center distance) tolerance combinations.

This paper presents the Direct Gear Design fillet profile optimization technique, which allows for a substantial bending stress reduction in comparison to traditionally designed gears. It also describes how bending stress reduction can produce other gear performance benefits.

**OPTIMIZATION METHOD**

Direct Gear Design [4] defines all gear geometry parameters without using the pre-selected basic or generating rack. It is applied for custom gears and allows separating the active involute flank and tooth fillet design. The flank profiles are designed first to satisfy primary performance requirements, such as maximum...
load stress level, maximum gear mesh efficiency (minimum specific sliding), etc.

The tooth fillet design is based on completely defined involute flank parameters. The initial fillet profile is a trajectory of the mating gear tooth tip in the tight (zero backlash) mesh. For practical purpose, this trajectory is defined at the minimum center distance (including both of the gear’s runout), maximum tooth thickness, and maximum outer diameter of the external mating gear (for internal mating gear the minimum inner diameter is used). This allows excluding interference with the mating gear tooth. The fillet optimization was based on three major components [5]: random search method locating fillet points; trigonometric functions for fillet profile approximation; and FEA for stress calculation.

The first and the last fillet points of the initial fillet profile lay on the form diameter circle (fig. 2) and cannot be moved during an optimization process. The random search method is moving the fillet nodes along the beams that pass through the fillet center and the nodes of the initial fillet profile. The center of the fillet is the center of the best-fitted circle. The bending stresses are calculated for every new fillet point’s combination. If the maximum bending stress is reduced, the program continues searching in the same direction. If not it steps back and starts searching the different direction. The number of iteration steps (or optimization time) is limited. Extensive testing of this program allowed defining the set of random search parameters that provides satisfying solutions for all possible combinations of gear parameters. The random nature of this method does not get repeatedly absolutely identical results for the same set of gear parameters and number of iteration steps. The program was adjusted to provide the maximum bending stress difference for repeatable calculation not to exceed 2 percent. The fillet shapes for these cases are also slightly different.

Fig. 4: Bending stress distribution chart along the fillet profiles. (1) fillet profile (black) generated by the standard (for coarse diametral pitch) rack with the tip radius 0.3/P; (2) fillet profile (pink) generated by the standard (for fine diametral pitch) rack with the tip radius equal zero; (3) fillet profile (dark blue) generated by the full tip radius rack; (4) circular fillet profile (light blue); (5) optimized fillet profile (green).
OPTIMIZATION RESULTS

As an example of the fillet profile optimization, different fillets were constructed for the gear pair with the standard involute tooth profile and the following parameters (fig. 3):

- Number of teeth of both mating gears—24
- Diametral pitch—12
- Generating rack profile (pressure) angle—20°
- Addendum coefficient—1.0
- Face width of both mating gears—1.0”
- Operating torque—200 in-lb.

Table 1 presents of the FEA of the different fillet profiles. It also indicates that the optimized fillet has the largest curvature radius at the maximum stress point and the shortest radial distance from this point to the load application point.

Figure 4 shows the bending stress distribution along the different fillet profiles. It clearly indicates that the optimized fillet has the lowest maximum bending stress, which is evenly distributed along the large portion of the fillet profile. Other fillet profiles have significantly greater maximum stresses that are sharply concentrated.

BENEFITS OF FILLET OPTIMIZATION

The graphs in Fig. 5 present stresses and efficiency of the gears with different fillets, the constant center distance \( a_w = 60 \text{ mm} \), the face width of both gears \( b = 10 \text{ mm} \), and the driving torque \( T = 50 \text{ Nm} \). The graphs show that, if the gears with the standard and optimized fillet have the same acceptable bending stress level, the gears with optimized fillet have finer pitch (smaller module) and higher number of teeth. This results with contact stress reduction because of the increased contact ratio and increased mesh efficiency.

<table>
<thead>
<tr>
<th>Fillet profile # at Fig. 3</th>
<th>Rack Cutter with tip radius R=0</th>
<th>Rack Cutter with tip radius R=0.3/P</th>
<th>Rack Cutter with full tip radius</th>
<th>Circular Fillet profile</th>
<th>Optimized Fillet profile</th>
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<tr>
<td>X-coordinate of load application point, in</td>
<td>-.0593</td>
<td>-.0593</td>
<td>-.0593</td>
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<td>Y-coordinate of load application point, in</td>
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<tr>
<td>X-coordinate of max. stress point, in</td>
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<td>-.0858</td>
<td>-.0868</td>
<td>-.0822</td>
<td>-.0813</td>
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<td>Y-coordinate of max. stress point, in</td>
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<td>.9026</td>
<td>.9026</td>
<td>.9158</td>
<td>.9234</td>
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<td>Fillet curvature radius at the max. stress point, in</td>
<td>.0231</td>
<td>.0399</td>
<td>.047</td>
<td>.0483</td>
<td>.1093</td>
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<td>Radial distance between load application and max. stress points, in</td>
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<td>0.1117</td>
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<td>Radial clearance, in</td>
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<td>.0159</td>
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<td>Max. Bending Stress, psi</td>
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<td>6602</td>
<td>6412</td>
<td>5731</td>
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<td>Relative stress difference, %</td>
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<td>0</td>
<td>-9.4</td>
<td>-12.0</td>
<td>-21.4</td>
</tr>
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</table>

Table 1: FEA of the different fillet profiles.
**REFERENCES:**


**Fig. 5:** (note to designer: labeled as fig. 4 in original document) (A) contact stress reduction; (B) increased mesh efficiency.

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