This paper presents a unique methodology for designing gears to enhance strength and life while allowing size and wear reduction. This new approach, trade named Direct Gear Design (DGD), optimizes the gear geometry to impart superior drive performance versus traditional gear design methods. This paper explains this alternative approach and demonstrates its effectiveness for high performance gear transmissions.

Introduction

Traditional standard gear design [1] is based on basic (or generating) gear rack. Parameters of this basic gear rack such as a profile angle, addendum, whole depth, and fillet radius proportions were standardized and used as input gear design parameters. The 20° pressure angle or other basic gear racks became a starting point and foundation of gear design. This made gear design indirect depending on pre-selected, typically standard, basic gear rack parameters. Major benefits provided by traditional gear design approach are gear interchangeability and low tooling inventory. For better gear performance (higher efficiency, load capacity, etc.) there are addendum modifications (or X-shifts) recommended by standards. However, selection of the standard basic rack imposes certain limitations on gear performance because, in this case, all possible gear solutions are inside the so-called block-contours [2]. In order to improve gear performance beyond the block-contour limit, the non-standard basic rack or individual basic racks for the mating gears should be considered. Besides, there is a clear trend to gear customization and interchangeability of gears is not very critical anymore. Low tooling inventory is not that important, either. Leading gear applications (aerospace and automotive, for example) do not use the standard basic racks for gear design, replacing them with the custom racks (with increased pressure or contact ratio, for example), providing higher gear performance.

The gear generating fabrication process (or hobbing), which uses the generating rack as a cutting edge of the tool, is not so dominating any more in gear production. Gears are also made by profile cutting, grinding, broaching, etc. There are many gear forming technologies, such as precision forging, casting, and extrusion, powder metal processing, plastic and metal injection molding, etc. All these technologies do not use the gear generating method. However, they are used to make gears, traditionally designed based on pre-selected gear racks. The proposed alternative Direct Gear Design method does not use the basic rack parameters. It uses desired performance parameters and operating conditions to define gear shape.

The idea of Direct Gear Design is not new. Ancient engineers successfully used it centuries ago. They used desired gear performance and known operating conditions to define gear geometry. Then they made gear drives to this geometry using available materials, technology and tools. It is important to note that the gear geometry was defined first. In other words gear parameters were primary, and the manufacturing process and tool parameters were secondary. This is the essence of Direct Gear Design.

This design approach is developed for involute gears and based on the Theory of Generalized Parameters created by Prof. E.B. Vulgakov [3] and can be defined as an application driven gear drive development process with primary emphasis on performance maximization and cost efficiency without concern for any predefined tooling parameters.

Gear Tooth and Mesh Synthesis

There is no need for a basic (or generating) gear rack to describe the gear tooth profile. Two involutes of the base circle, the arc distance between them, and tooth tip circle describe the gear tooth (Fig.1). The equally spaced teeth form the gear. The fillet between teeth is not in contact with the mating gear teeth. However, this portion of the tooth profile is critical because this is the area of the maximum bending stress concentration.

Two (or more) gears with the equal base circle pitch can be put in mesh (Fig.2). The operating pressure angle α₀ and the contact ratio ε₀ are defined by the following formulae [3, 4]:

For external gearing
\[ \alpha_0 = \arccos \left( \frac{d_2}{d_1} \right) \]
\[ \varepsilon_0 = \frac{n_2}{n_1} - \frac{1}{u} \]

For internal gearing
\[ \alpha_0 = \arccos \left( \frac{d_1}{d_2} \right) \]
\[ \varepsilon_0 = \frac{n_2}{n_1} + \frac{1}{u} \]

Where
\[ u = n_2 / n_1 \]

For metric system gears the operating module is \[ m_1 = 2 * a_0 / (n2 \pm n1) \]. For English system gears the operating diametral pitch is \[ p_1 = (n2 \pm n1) / (2 * a_0) \]. The “+” is for the external gearing and the “-” is for the internal gearing.
Tooth Fillet Profile Design and Optimization

In order to complete the nominal gear geometry description, the tooth fillet profile should be defined. In traditional gear design the fillet profile is a trajectory of the tool cutting edges in generating motion. The most common way to reduce bending stress concentration is using the full radius generating rack. In some cases the generating rack tip as formed by parabola, ellipse, or other mathematical curves. All these approaches have limited effect on bending stress reduction, which depends on the generating rack profile angle and number of gear teeth.

In Direct Gear Design the fillet profile is optimized in order to minimize bending stress concentration. The initial fillet profile is a trajectory of the mating gear tooth tip in the tight (zero backlash) mesh. The FEA and random search method are used for fillet optimization [5]. The calculation process results with forming the optimized fillet profile that provides minimum achievable bending stress. This fillet provides the minimized radial clearance with the mating gear tooth, excluding interference at worst tolerance combination and operating conditions. It also has the maximized curvature radius, distributing the bending stress along a large portion of the fillet, reducing stress concentration (Fig. 3). The shape of the optimized fillet profile depends on the mating gear geometry. It would be very different if the same gear were in mesh the external gear, or the internal gear, or the gear rack. However, it practically does not depend on the load level and load application point. If the gear is in mesh with several different gears, like in the planetary stage for example, its profile is optimized to exclude interference with any gear.

Table 1 presents bending stress reduction, achievable by the full radius rack application and by the fillet profile optimization in comparison to the standard 20°.

FIG 2: GEAR MESH; a—EXTERNAL GEARING; b—INTERNAL GEARING; aW—CENTER DISTANCE; pb—BASE CIRCLE PITCH; αw—OPERATING PRESSURE ANGLE; εα—CONTACT RATIO; dW1,2—OPERATING PITCH CIRCLE DIAMETERS; SUBSCRIPTS “1” AND “2” ARE FOR THE MATING PINION AND THE GEAR.

FIG 3: BENDING STRESS DISTRIBUTION ALONG THE FILLET; a—THE STANDARD AGMA 201.02 20 DEGREE RACK GENERATED FILLET; b—THE FULL RADIUS 20 DEGREE RACK GENERATED FILLET; c—THE OPTIMIZED FILLET; d—THREE FILLET OVERLAY.
and 25° rack for gears with different number of teeth. The involute portion of the tooth profile is the same.

The graphs in Fig. 4 present stresses and efficiency of the gears with different fillets, the constant center distance $a_w = 60$ mm, the face width of both gears $b = 10$ mm, and the driving torque $T = 50$ Nm. The graphs show that, if the gears with the standard and optimized fillet have the same acceptable bending stress level, the gears with optimized fillet have finer pitch (smaller module) and higher number of teeth. This results with contact stress reduction, because of the increased contact ratio and increased mesh efficiency.

**Bending Stress Balance**

Mating gears should be equally strong. If the initially calculated bending stresses for the pinion and the gear are significantly different, the bending stresses should be balanced [5].

DGD defines the optimum tooth thickness ratio $S_{w_1}/S_{w_2}$ (Fig. 3), using FEA and an iterative method, providing a bending stress difference of less than 1 percent. If the gears are made out of different materials, the bending safety factors should be balanced.

Direct Gear Design is applicable for all kind of involute gears: the spur gears including, external, rack and pinion, and external, helical, bevel, worm, face gears, etc. The helical, bevel, and worm gear tooth profile is typically optimized in the normal section. The face gear fillet is different in every section along the tooth line. Therefore its profile is optimized in several sections and then is blended into the fillet surface.

**Gears with Asymmetric Teeth**

Two profiles (flanks) of a gear tooth are functionally different for many gear types and 25° rack for gears with different number of teeth. The involute portion of the tooth profile is the same.

The graphs in Fig. 4 present stresses and efficiency of the gears with different fillets, the constant center distance $a_w = 60$ mm, the face width of both gears $b = 10$ mm, and the driving torque $T = 50$ Nm. The graphs show that, if the gears with the standard and optimized fillet have the same acceptable bending stress level, the gears with optimized fillet have finer pitch (smaller module) and higher number of teeth. This results with contact stress reduction, because of the increased contact ratio and increased mesh efficiency.

**Bending Stress Balance**

Mating gears should be equally strong. If the initially calculated bending stresses for the pinion and the gear are significantly different, the bending stresses should be balanced [5].

DGD defines the optimum tooth thickness ratio $S_{w_1}/S_{w_2}$ (Fig. 3), using FEA and an iterative method, providing a bending stress difference of less than 1 percent. If the gears are made out of different materials, the bending safety factors should be balanced.

Direct Gear Design is applicable for all kind of involute gears: the spur gears including, external, rack and pinion, and external, helical, bevel, worm, face gears, etc. The helical, bevel, and worm gear tooth profile is typically optimized in the normal section. The face gear fillet is different in every section along the tooth line. Therefore its profile is optimized in several sections and then is blended into the fillet surface.
drives. The workload on one profile is significantly higher and is applied for longer periods of time than for the opposite one. The design of an asymmetric tooth shape reflects this functional difference. The gears with asymmetric teeth (Fig. 6) are naturally suitable for Direct Gear Design because there are not standards for the asymmetric basic or generating racks. The design intent of asymmetric gear teeth is to improve performance of the primary drive profiles by some degrading performance of the opposite coast profiles. The coast profiles are unloaded or lightly loaded during relatively short work period.

The main advantage of asymmetric gears is contact stress reduction, resulting higher torque density (load capacity per gear size). Another important advantage is possibility to design the coast flanks and fillet independently from the drive flanks, managing tooth stiffness, and load sharing, while keeping a desirable pressure angle and contact ratio on the drive profiles. This allows reducing gear noise and vibration level.

The DGD approach for asymmetric gears is the same as for symmetric gears. The only difference is that the asymmetric tooth (Fig. 7) is defined by two involutes of two different base circles \( d_{bd} \) and \( d_{bc} \). The common base tooth thickness does not exist in the asymmetric tooth. The circular distance (tooth thickness) \( S_p \) between involute profiles is defined at some reference circle diameter \( d_i \) that should be bigger than the largest base diameter.

The asymmetric gear mesh (Fig. 8) presents two different drive and coast flank meshes.

![FIG. 6: GEAR WITH ASYMMETRIC TEETH](image1)

![FIG. 7: ASYMMETRIC TOOTH PARAMETERS](image2)

![FIG. 8: ASYMMETRIC GEAR MESH](image3)

![FIG. 9: TOOLING PROFILES; A—PROFILE GEAR MACHINING, B—GENERATING GEAR MACHINING, C—GEAR MOLDING, POWDER METAL PROCESSING, OR CASTING.](image4)
with different pressure angles and contact ratios. With asymmetric gears it is possible to simultaneously increase the transverse contact ratio and operating pressure angle beyond the conventional gear limits.

Table 2 presents comparison traditionally designed gear pair with the bending stresses balanced by addendum modification, generated by the full radius 25° rack with similar gear pairs created by Direct Gear Design with different tooth profiles: symmetric, asymmetric, with conventional and high contact ratio. This table illustrates the contact and bending stress reduction as a result of complete tooth profile optimization.

**Tooling for Directly Designed Gears**

The optimized gear profiles require the custom tooling. For profile machining process (Fig. 9a) the tool profile is the same as a space profile between the neighboring teeth. For generating the tool profile is defined by reverse genera-
tion, when the designed gear forms the tooling rack profile. The pressure angle, in this case, is selected to provide better machining conditions. For gear molding, powder metal processing, and casting (Fig. 9c) the tool cavity profile is the same as the whole gear profile, adjusted for warpage and shrinkage.

Summary

Now gear designers can decide to use the traditional or the Direct Gear Design method. Traditional gear design is driven by manufacturing, when gear interchangeability, low tool inventory, and design simplicity are important. It is for low expectation gear drives. Typical examples are shelf gears, low production volume machined gears, gear drives with interchangeable gear sets, etc.

Direct Gear Design is driven by application, when technical and market performance of product is critical. It is for custom gear drives. Typical examples are the molded, forged, cast, and powder metal gears.

The mass production machined gears, gears for critical and extreme applications, like aerospace, automotive, and racing gear transmissions.
Conclusion

The presented alternative Direct Gear Design method provides complete gear tooth profile optimization resulting in significant contact and bending stress reduction. This stress reduction is converted to:

- Higher load capacity
- Reduced size and weight
- Extended lifetime
- Reduced noise and vibration
- Higher efficiency
- Higher reliability
- Reduced cost

About the Author:

Dr. Alexander L. Kapelevich is owner of the consulting firm AKGears, LLC. He has 30 years of experience in gear transmission development. He can be reached by e-mail at ak@akgears.com. AKGears has used Direct Gear Design for many years in developing high performance gear transmissions. For more information go to [www.akgears.com]. Reprinted from the Proceedings of Global Powertrain Congress 2007, Vol. 39-42, June 17-19, 2007, Berlin, Germany, [www.gpc-icfem.org], (734) 944-5850.

References: