METHODS TO DETERMINE FORM DIAMETER ON HOBBED EXTERNAL INVOLUTE GEARS
To assist gear designers to instantly verify the manufacturability of their designs, two methods are presented to calculate the manufactured form diameter given the design parameters of the gear and the hob.

By SHUO “WILL” ZHANG

In custom designs of external spur or helical gears, designers generally prefer a lower form diameter for greater transverse contact ratio. A larger root fillet radius and a higher root diameter are also preferred for higher root bending strength. The combination of the above three requirements can render the gear designs unable to be manufactured by hobbing process followed by flank-only finishing processes. To assist gear designers to instantly verify the manufacturability of their designs, two methods are presented in this work to calculate the manufactured form diameter given the design parameters of the gear and the hob. In particular, the second method does not involve iterative calculations or graphical simulations that are adopted by numerous publications and software programs. Instead, a series of empirical formulas are provided for users to directly find the manufactured form diameter using the gear design parameters and hob geometries. Over wide ranges of gear design parameters, the formulas can calculate form diameters with relative errors smaller than 0.1%. Examples of applying the formulas are also presented.

1 INTRODUCTION

For involute gears, form diameter is the diameter of a circle at which the root fillet curve intersects or joins the involute [1]. In most cases, the root fillet curve is one continuous trochoid produced by the tip of the generating cutting tool. In some cases, the root fillet curve can also consist of a trochoid and another involute at a lower pressure angle. Form diameter is also known as true involute form diameter (TIF), or diameter at start of involute (SOI) [2].

Designers of custom external involute gears usually specify a maximum form diameter, which is necessary for the transverse contact ratio of the gear mesh under various conditions. Meanwhile, a higher root diameter and a larger radius of the root fillet are also preferred for higher root bending strength. However, when the above three requirements are specified at the same time, they may cause challenges to the manufacturing of the gear, especially if a notch in fillet caused by the finishing operation is not allowed. If the gear designers can instantly determine whether the specified form diameter can be manufactured along with the specified root fillet radius and root diameter, gear designs that cannot be manufactured can be prevented.

A number of commercial software programs can simulate the manufacturing processes accurately. They usually present a graphical simulation of the generating cutting process, based on the parameters of the gears and cutting tools. However, if the gear design is not created using such programs, it can take additional cost and time to perform the manufacturing simulation with them.

Multiple methods have been presented in prior works to determine the form diameter. Gerpen and Reece presented a method with the simplifying assumption that the form diameter is generated by the end point of the circular tip [3]. Math and Chand presented a method for hobs that have two other straight segments connecting the circular tip and the main cutting flank of the hob [4]. Lian presented a method to determine form diameter of helical or spur, internal and external gears cut by either shaper cutters or hobs [5], if the root fillet consists of only a trochoid generated by the circular tip of cutter, without a secondary involute generated by the transitional straight edge next to the circular tip on the cutter. In addition, all the above methods involve solving a series of equations to determine the form diameter.

This paper presents two methods, A and B, to determine the manufactured form diameters on external helical or spur gears, produced by hobbing process, possibly followed by flank-only finishing process such as shaving or grinding. Method A involves equations to describe three key curves: the trochoid generated by the circular tip of the hob, the primary involute generated by the main cutting flank of the hob, and the secondary involute generated by the transitional straight edge connecting the circular tip and the main cutting flank on the hob. Then the lowest point on the primary involute is determined, which can be intersected by either the trochoid or the secondary involute. Method A involves iterative solving to find the intersection point, similar to previous publications.

Method B consists of a series of empirical formulas, and a flow chart showing how to apply the formulas, to calculate the form diameter without iteration. The formulas are found by regressing a large quantity of examples generated using method A, which consist of form diameters and gear design parameters and hob parameters. Due to the nature of empirical formula, form diameters from method B have relative errors, which are less than 0.1% over wide ranges of input parameters as specified, and are less than 0.05% for common gear designs. Tooth thickness is not needed in either method A or B. Examples of applying both methods are shown.
2 SYMBOL DEFINITIONS

The presented methods apply to external helical or spur gears produced by hobbing process, followed by flank-only finishing process. The methods are presented in section 3.1 and 3.2 assuming the gear is a spur. Section 3.3 discusses how to handle helical gears. An example of a gear tooth is shown in Figure 1.

The methods assume that the concerned geometries of the hob can be represented by Figure 2. In the normal plane of the hob, a circular tip with radius $r_f$ has two endpoints M and N, and a center point H. A transitional straight edge NQ is tangent to MN at point N, and joins main cutting flank PQ at point Q.

Both methods use the following input variables, which are assumed known to the designers:

\[ \theta_p \] Pertainning to the gear: $r_f$, $r_b$, $Z$, $\phi_f$

\[ \theta_b \] Pertainning to the hob: $\phi_b$, $\phi_2$, $\phi_0$

In method A, the pressure angle of NQ, $\phi_2$, can vary from $0^\circ$ to $\phi_b$. In method B, for simplicity purposes, $\phi_2$ is assumed to be $10^\circ$, therefore is not an input variable. If the hob does not have protuberance, $\phi_0 = 0$, and $\phi_2 = \phi_b$.

If $\phi_0$ is the actual protuberance on the hob, the calculated form diameter corresponds to the condition of the gear after hobbing and before finishing. If the form diameter of the gear after flank-only finishing operation is to be calculated, $\phi_0$ needs to be replaced with $\phi_{a0}$, before applying either method:

\[ \phi_{a0} = \phi_0 - u_s \cos \phi \]  \hspace{1cm} \text{Equation 1}

in which $u_s$ is the flank-only finishing operation stock removal along the generating pitch circle of the hobbing process.

In custom gear designs, the pressure angle of the gear is not an essential parameter. The same gear can be expressed using various

Table 1: Symbols used in method A.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Terms</th>
<th>Units</th>
<th>First used</th>
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</thead>
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<tr>
<td>$\phi$</td>
<td>Hob normal pressure angle</td>
<td>rad in A, deg in B</td>
<td>Eq (1)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>Hob secondary pressure angle for tip transition</td>
<td>rad in A, deg in B</td>
<td>Eq (A8)</td>
</tr>
<tr>
<td>$r_{f0}$</td>
<td>Hob tip radius</td>
<td>in or mm</td>
<td>Eq (A1)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>Angle between centerline of H and starting point S of involute PQ</td>
<td>rad</td>
<td>Eq (A5)</td>
</tr>
<tr>
<td>$a_2$</td>
<td>Angle between centerline of H and starting point S of involute NQ</td>
<td>rad</td>
<td>Eq (A6)</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>Helix angle of gear at base circle</td>
<td>rad</td>
<td>Eq (3)</td>
</tr>
<tr>
<td>$\beta_c$</td>
<td>Helix angle of gear at generating pitch circle</td>
<td>rad</td>
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Table 2: Symbols used only in Method B.

<table>
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<td>Form diameter, if produced by the actual hob circular tip</td>
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<td>Figure 2</td>
</tr>
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<td>$d_{HC0}$</td>
<td>Form diameter, if produced by a sharp hob tip, $C = 0$</td>
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<td>Figure 3-5</td>
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<tr>
<td>$d_{HC2}$</td>
<td>Form diameter, if produced by a hob tip with $C = 0.0005$</td>
<td>in or mm</td>
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<tr>
<td>$d_{HC3}$</td>
<td>Form diameter of gear, if produced by the secondary straight edge NQ of the hob</td>
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<td>Figure 11</td>
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<td>$i, j, k, m$</td>
<td>Intermediate variable to determine which segment of the hob produces form diameter</td>
<td>--</td>
<td>Figure 5</td>
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</table>

Table 3: Description of key points and curves in figures.

3 METHODS TO DETERMINE FORM DIAMETER

Figure 1: Example of a gear tooth.

Figure 2: Geometries of a hob.
pressure angles. Therefore, pressure angle of the gear, normal or transverse, is not an input variable in either method. Normal pressure angle of the hob is important and does not need to equal pressure angle of the gear. Note that the variable \( r \) is the generating pitch radius of the gear, which can be found using equation 2 for spur gear:

\[
r = r_b \frac{\cos \phi}{\cos \phi}
\]

Equation 2

For helical gear, generating pitch radius \( r \) can be found by solving equation 3:

\[
\frac{r}{r_b} \cos \left( \tan^{-1} \left( \tan \phi \sqrt{1 + \left( \frac{r}{r_b} \tan \beta_b \right)^2} \right) \right) = 1
\]

Equation 3

### 3.1 METHOD A

In method A, three key curves need to be expressed analytically. The trochoid MN' generated by the circular tip of the hob is expressed using equation A1-A4 \([6]\), the primary involute PQ' generated by the main cutting flank of the hob, and the secondary involute NQ' generated by the transitional straight edge connecting the circular tip and the main cutting flank on the hob. To space the three curves in the correct relations to each other, the angular spacing from centerline of the trochoid MN' to both involute curves is given in equation A5 and A6.

Then the lowest point X on the primary involute PQ' is determined, which can be intersected by either the trochoid MN' or the secondary involute NQ'. Method A involves iterative solving to find the intersection point, similar to previous publications.

During the hobbing process, in the reference frame of the gear, the hob tip center H has a trajectory H', which is symmetric to its centerline, as shown in Figure 3 and 4. Trochoid curve H can be expressed using variables \( \omega_T \) and \( r_T \):

\[
\theta_T = \tan^{-1} \left[ \frac{r_T^2 - (r_T + \rho_{ao})^2}{r_T + \rho_{ao}} \right] - \frac{\sqrt{r_T^2 - (r_T + \rho_{ao})^2}}{r}
\]

Equation A1

The envelope curve MN' of circular tip MN is an equidistant curve from H' with a spacing equal to the hob tip radius \( \omega_{ao} \). Therefore MN' can be expressed using equation A2-A4:

\[
\psi_T = \tan^{-1} \left[ \frac{r (r_T + \rho_{ao}) - r_T^2}{\sqrt{r_T^2 - (r_T + \rho_{ao})^2}} \right]
\]

Equation A2

\[
r_T = \sqrt{r_T^2 + \rho_{ao}^2 - 2 r_T \rho_{ao} \sin \psi_T}
\]

Equation A3

\[
\theta_T = \theta_T + \cos^{-1} \left( \frac{r_T - \rho_{ao} \sin \psi_T}{r_T} \right)
\]

Equation A4

The angle between centerline of H' and starting point of the involute PQ' is given by equation A5:

\[
\alpha = \frac{(r - r_T) \sin \phi + \rho_{ao} (1 - \sin \phi) - \delta_0}{r_b} - \tan \phi + \phi
\]

Equation A5

Similarly, the angle between centerline of H' and starting point of involute NQ' is given by equation A6:

\[
\alpha_2 = \frac{(r_b / \cos \phi_2 - r_T) \sin \phi_2 + \rho_{ao} (1 - \sin \phi_2)}{r_b} - \tan \phi_2 + \phi_2
\]

Equation A6

There are multiple possibilities of the relative positions of the three curves. Nemcek and Dejl made an in-depth analysis of them \([7]\). Figure 5 presents two representative conditions, and where the form diameter is in those conditions. To find the lowest point X on the primary involute, it is recommended to first iteratively find the intersection point \( X_0 \) of the primary involute PQ' and the root trochoid MN'. Make sure to find the higher one when there are two intersection points. Then determine whether the segment of NQ' on
the outside of MN’ has an intersection point with PQ’. If so, the new intersection point will be higher than X₀ and becomes the lowest point X on the primary involute, as shown in Figure 5(b). If not, X₀ is the lowest point X on the primary involute, as shown in Figure 5(a).

When the iterations are fully converged, form diameter from Method A is very accurate. They have been compared to results from KISSsoft using multiple examples, and almost all visible digits match. See section 4 for example details.

### 3.2 METHOD B

A large quantity of data sets is generated using method A, covering wide ranges of input parameters. Each data set includes input parameters and calculated form diameter. These data are then used in regression to create a series of empirical formulas to directly calculate form diameter from input parameters, without the need to analytically plot multiple curves and use mathematical relations to identify where the lowest point on primary involute is. Method B is believed to be much simpler to adopt.

Due to the nature of empirical formula, form diameters from method B have relative errors, which are generally less than 0.1% over the specified ranges of input parameters, and are less than 0.05% for common gear designs. The accuracy is good enough for most practical use, especially when designers need to verify the manufacturability of specified form diameter, root diameter and hob tip radius.

The 18 equations should be applied in the sequence shown in Figure 6. 135 coefficients \( d_{ijk} \) are provided in Table 4, which are used to calculate 45 coefficients \( c_{ijk} \), which are later used to calculate 15 coefficients \( b_i \), which are later used to calculate 5 coefficients \( a_i \), which are later used to calculate candidate form diameter \( d_{TIF-C} \). Depending on the input parameters and the logical judgment shown in Figure 6, similar processes may also be needed to find the candidate form diameter when \( C = 0.0005, d_{TIF-CE}, \) and candidate form diameter when protuberance is zero \( d_{TIF-CD}, \) and the candidate form diameter if it is produced as the intersection of two involute curves, \( d_{TIF-S} \).

Finally, the method will determine which candidate form diameter is the actual form diameter and report it as \( d_{TIF} \). Although method B takes 75 to 178 calculation steps to determine form diameter, all steps are simple and direct.

To have a relative error less than 0.1%, variables \( \varepsilon, A, B, C \) need to be within the given ranges below:

- \( \varepsilon \): \([15°, 30°]\)
- \( A \): \([0.01, 0.2]\)
- \( B \): \([0, 0.6]\)
- \( C \): \([0, 0.005]\)

The equations for method B are below:

- \( A = \frac{r - r_f}{2r} \)  
  \( \text{Equation B1} \)
- \( B = \frac{\rho_{ao}}{r - r_f} \)  
  \( \text{Equation B2} \)
- \( C = \frac{\delta_2}{2r} \)  
  \( \text{Equation B3} \)
- \( G = C + A B \cos \phi \left[ 0.8391 - \tan \left( \frac{90 - \phi}{2} \right) \right] \)  
  \( \text{Equation B4} \)
- \( 1f \ \phi > 18, \quad G_C = 0.0065 \)  
  \( \text{Equation B5.1} \)
- \( 1f \ \phi \leq 18, \quad G_C = 0.001 \phi - 0.0115 \)  
  \( \text{Equation B5.2} \)
- \( c_{ijk} = \sum_{m=0}^{2} d_{ijkm} \phi^m \)  
  \( \text{Equation B6} \)
- \( b_{ij} = \sum_{k=0}^{2} c_{ijk} C^{k/2} \), except for \( i = j = 0 \)  
  \( \text{Equation B7.1} \)
- \( b_{00} = \sum_{k=0}^{2} c_{00k} C^k \)  
  \( \text{Equation B7.2} \)
- \( a_i = \sum_{j=0}^{2} b_{ij} B^j \)  
  \( \text{Equation B8} \)
- \( g = 0.001 \sin \left( \frac{\phi - 16}{7} \pi \right) e^{-\left| \frac{25A - \phi}{6} + 2.79 \right|} \)  
  \( \text{Equation B9} \)
- \( d_{TIF-C} = 2 \ r_y \left( 1 + g \right) \sum_{i=0}^{4} a_i A^i \)  
  \( \text{Equation B10} \)
- \( b_{ij} = \sum_{k=0}^{3} d_{ijk} \phi^k \)  
  \( \text{Equation B11} \)
Coefficients used in equation B6 are shown in Table 4.
Coefficients used in equation B11 are shown in Table 5.
Coefficients used in equation B15 are shown in Table 6.

### Table 4: \( d_{ijk} \) used in equation B6.

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<th>( i = 4 )</th>
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### Table 5: \( d'_{ijk} \) used in equation B11.

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### Table 6: \( d_{ijk} \) used in equation B15.

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### 3.3 Applying Method A & B to Helical Gears

Method A or B cannot be applied in the transverse plane to accurately determine the form diameter of a helical gear, due to the assumption by both methods that the hob tip is circular. When a hob has a circular tip in its normal plane, the tip has an elliptical cross section.
in the transverse plane when cutting a helical gear.

Both methods can be applied to helical gears in the normal plane using the concept of virtual spur gear. The generating pitch circles of the helical gear and its virtual spur gear intersect at point \( R \) in Figure 3. However, if Equation 4 is used for the number of teeth in the virtual spur gear, the form diameter results can have relative errors greater than 0.1%. A slightly different method to calculate the number of teeth in the virtual spur gear \( Z_v \) is using Equation 5, which provides higher fidelity of the generated root geometry for a wide variety of gear designs. The relative error of helical gear form diameter using Equation 5 and method A is less than 0.01%. The relative error of helical gear form diameter using Equation 5 and method B is mainly from the method itself, which is less than 0.1% if design parameters are in the specified ranges.

\[
Z_{V,I} = \frac{Z}{(\cos \beta_r)^2} \quad \text{Equation 4}
\]

\[
Z_V = \frac{Z}{(\cos \beta_r)^1.5 \, (\cos \beta_{TIF,I})^{1.4}} \quad \text{Equation 5}
\]

Helix angle at various diameters can be related to base helix angle using equation 6.

\[
\tan \beta_b \div 2 \, r_b = \tan \beta_r \div 2 \, r = \tan \beta_{TIF,I} \div d_{TIF,I} \quad \text{Equation 6}
\]

The center distance between helical gear and its virtual spur gear is equal to the difference between the generating pitch radius of the two gears:

\[
E = \frac{Z}{Z} \, r \cos \beta_r - r \quad \text{Equation 7}
\]

In which \( r \) and \( \beta_r \) can be found using equation 3.

After form diameter of the virtual spur gear \( d_{TIF,V} \) is found using either method A or B, center distance adjustment is needed to find form diameter of the actual helical gear using equation 8:

\[
d_{TIF} = d_{TIF,V} - 2E \quad \text{Equation 8}
\]

It is recommended to follow the process shown in Figure 7 to calculate form diameter of a helical gear.

4 EXAMPLES USING METHOD A AND B

Ten examples are created to compare form diameter results from method A, method B to KISSsoft. Each example represents a rather extreme feature of the gear design, as noted in the description, with the hope of covering a wide range of practical gear designs. Dedendum, large protuberance, large hob tip radius, and large secondary pressure angle tend to cause the main involute to be intersected by the secondary involute rather than the trochoid.

By applying either method multiple times with various protuberances ranging from zero to the actual protuberance, one can construct the shape of the upper portion of the root fillet curve, from form diameter to maximum undercut diameter or base circle diameter, whichever is greater. The shape of the fillet curve can be useful for the design of the tool for finishing operations.

6 CONCLUSIONS AND FUTURE WORK

Two methods are presented to determine form diameter of external helical or spur gears from gear design parameters and hob parameters. Method A can be very accurate but involve some difficulty to apply. With method A, one needs to iteratively find one particular intersection point out of a few intersection points among three curves which are expressed with implicit functions.

In comparison, method B is easier to apply, as all equations and steps are direct calculations. Method B has greater relative errors which are acceptable for most practical purposes. One possible use of such methods is that gear designers without access to advanced simulation software can evaluate manufacturability of proposed designs, and correlate achievable contact ratio to root radius and critical section tooth thickness early in the design stage. Other uses of the methods include assisting hob design and performing parametric study of gear and hob parameters, which is time-consuming even with access to advanced simulation software.

For future work, such methods can be expanded to account for internal gears and shaper-cut gears.
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<th>( \rho_{b0} )</th>
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Table 7: Examples of applying method A and B.
**ACKNOWLEDGEMENT**

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**BIBLIOGRAPHY**


**ABOUT THE AUTHOR**

Shuo “Will” Zhang is the Advanced R&D Engineer at Dana Off-Highway Lafayette IN (formerly Oerlikon Fairfield). Zhang holds MSME and BSME degrees from Purdue University and a BSME degree from Shanghai Jiaotong University. At his seventh year with Dana, he is responsible for the design and research of off-highway power transmissions and related advanced concepts. His expertise focuses on simulation and validation of the mechanical and thermal durability, efficiency, and NVH of drive products. Zhang is a member of the AGMA Epicyclic Enclosed Drives (Planetary) Committee and Lubrication Committee.