INTEGRATING NON-CONTACT METROLOGY IN THE PROCESS OF ANALYSIS AND SIMULATION OF GEAR DRIVES
The application of non-contact metrology allows a very fast collection of points on the measured gear-tooth surfaces with data rates that can be as high as millions of points per second — this new technology provides a wealth of information on gear geometry.

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Speed and accuracy are critical factors for gear inspection. The faster the inspection process, the larger number of gear units can be inspected. The accuracy of the inspection process also plays a fundamental role to identify any possible defect in the production line and before the gearbox is assembled and further tested.

The technology of non-contact metrology of gear drives has been developed during the last decade and it is currently available to be integrated, not only in the process of inspection of gears, as primarily intended, but in the process of analysis and simulation of gear drives as proposed here. This technology has come to stay and to change not only the way in which gear inspection and quality assessment of gears are performed, but to contribute to the process of reverse engineering of gear geometry, noise-root-cause analysis, or to serve as a reference for the determination of the baseline stress information for further gear design optimization as proposed in this paper.

The actual machines for non-contact metrology of gear drives allow for a fast measurement with high precision of all types of gear geometries, including that of the most complex shapes and gear types as hypoid and spiral bevel gears. This technology enables surface point clouds to be acquired at rates as large as 120,000 points per second taking, for example, only five minutes to have all tooth surfaces of a medium size gear inspected. The high-density acquisition of points with a point pitch of 20 µm enables not only the most accurate analysis of surface geometries but also reveals the smallest imperfections on the gear tooth surfaces such as machining traces and resulting surface waviness that will cause undesired vibrations or high frequency noise.

In this paper, a preliminary overview of the possibilities of integration of non-contact metrology in the process of analysis and simulation of gear drives will be provided. For that, two similar gears, with same macro-geometry characteristics and henceforth referred to as Gear A and Gear B, will be simulated based on their corresponding point clouds obtained by a non-contact metrology machine. Unfortunately, the gear quality was not known and no further information apart from the point clouds was disclosed for this work. Along the following sections, the process that has been followed to model the gears from the corresponding point clouds and the results obtained from the tooth contact analysis during their mesh with a common master (theoretical) gear will be presented. Finite element analysis will also be performed for both gears and the obtained results in terms of contact pressure and bending stresses shown.

**REGENERATING THE ACTIVE TOOTH SURFACES**

The active tooth surfaces of a gear tooth are those parts of the tooth surface that may come into contact with the mating gear tooth surfaces and therefore they are the parts of the tooth surface wherein the contact pattern is obtained.

Figure 1 shows a raw representation of the point cloud of the left side of gear tooth #1 of Gear A as obtained with a non-contact metrology machine. The cloud surface consists in more than 500,000 points for each side of the gear tooth surface. The mathematical processing of this point cloud for its integration in the process of analysis and simulation of the gear drive is computationally challenging. No information of the inspected gear was provided. The number of teeth and the hand of helix were obtained from the simple inspection of the set of point clouds for
all teeth of the gear. A custom-made module has been developed within IGD – Integrated Gear Design – software for the processing of point clouds obtained by non-contact metrology machines. Point cloud data consist in a list of coordinates x, y, and z of all the points.

Figure 2 represents the radial projection of the left side of the gear tooth surface on a plane containing the axis of rotation of the gear. The radial projection is a representation of the axial coordinate of the gear tooth surface, usually the coordinate z of the points, in the X-axis, and the radius of each point of the point cloud in \((\sqrt{x^2 + y^2})\) the Y-axis. It represents the 2D boundaries of the gear tooth surfaces. From the radial projection, the face-width and addendum radius of the inspected gear can be obtained. Moreover, the root radius can be also roughly determined and therefore, with this information, the module of the gear will be estimated as proposed in Section 4.

From Figure 2 and using the radial projection of the point cloud as reference, the intended face width is obtained to be 29.00mm, although for simulation purposes, the point cloud will be limited to a face width of 27.3652 mm. Those points near the edges of the surface will reduce the accuracy of the regenerated gear tooth surfaces and therefore they will be discarded.

In order to clean up those points near the edges of the surfaces, the boundaries of the active tooth surface and fillet will be considered on the radial projection of the tooth surface. Figure 2 shows the five limiting boundaries of the active surface and fillet portion of the tooth surface. The active surface of the tooth is limited by the top, bottom, front, and back boundaries. The fillet will be limited by the bottom, root, front and back boundaries, as shown in Figure 2. Points outside these boundaries are discarded. Using the intersection points of the boundaries, a theoretical regular grid is depicted on the radial projection of the gear tooth surface (see Figure 3). A switch button allows the user to transition between the 3D view and the radial projection of the gear tooth surface (2D view) at any time. The theoretical regular grid has to become an actual best-fit regular grid on the point cloud. The high density of the obtained point cloud allows the actual grid to be kept very close to the theoretical regular grid. This is necessary to obtain the best-fit NURBS surface to the point cloud. NURBS (Non-Uniform Rational B-Splines) are very appropriate to regenerate gear tooth surfaces from point clouds because this form of analytical surface allows partial derivatives to be obtained with respect to two surface parameters. Once the surfaces are regenerated by NURBS, these surfaces can be further used within the algorithms of tooth contact analysis or finite element modeling.

Figure 3 shows different densities for the theoretical and actual regular grid on the radial projection of the gear tooth surfaces. The theoretical regular grid is represented in blue color and the actual regular grid is represented overlaid in red color. As shown, the actual regular fit matches very well the target theoretical grid. Figure 3(a) considers a regular grid of 15 x 10 points (15 points in longitudinal direction and 10 points in profile direction) whereas Figure 3(b) shows a regular grid 30 x 20 points and Figure 3(c) shows a regular grid of 50 x 30 points. The chosen regular grid has to be defined only one time and it is kept for the analysis of the point cloud corresponding to any tooth of the gear or side of the gear tooth surfaces. The effect of the grid size on the results of tooth contact analysis and finite element analysis will be shown later in this paper.

The use of an efficient algorithm to find the nearest neighbor point to each theoretical regular grid node is very important mainly when dealing with data structures larger than half a million points. The authors’ approach is based on the use of a kd-tree, a data structure for storing a finite set of points from a k-dimensional space [1], and invented by Jon Bentley in the 1970s.

Once the regular grid based on actual points of the point cloud
is defined, a NURBS surface is created from the regular grid of 3D point coordinates. NURBS is one of the most employed surface fitting models, because it provides a standard representation of curves and surfaces [2] and it is widely supported by modern standards such as OpenGL and IGES, which are used for graphics and geometric data exchange [3]. In addition, the NURBS surface model has stability, flexibility, local modification properties and is robust to noise. However, the NURBS surface model needs the input data points mapped on a regular grid structure. Because point clouds from non-contact metrology machines are high density point clouds, obtaining regular grid structures is relatively straightforward independently of the grid size chosen.

Figure 4 shows the NURBS surface obtained from one of the regular grids shown in Figure 3 and represented together with the defining point cloud. The NURB surface interpolates the point clouds accurately. The same process described above has to be followed to regenerate the right side of the gear active tooth surface from the corresponding point cloud.

**REGENERATION OF THE FILLET**

The fillet is regenerated using Hermite curves. The application of Hermite curves in fillet gear modeling allows for the design of smooth curves based on a small number of user-controlled parameters, usually the initial and final tangent directions and weights of the curve [4]. The initial and final tangent directions are considered as known. The initial tangent direction can be obtained as the tangent to the NURBS active surface along the boundary separating it from the fillet. This boundary, although is unknown, has to be tentatively defined by the user (see the set-up boundaries in Figure 2). The final tangent direction is considered tangent to the root circle of the gear and therefore it is known (see Figure 5). The initial and tangent weights are obtained by an optimization process to minimize the deviations between the regenerated fillet surface based on Hermite curves and the point cloud.

Figure 6 shows the point clouds used for regeneration of gear tooth surface prior to the application of the described approach (left) and after the gear tooth surfaces are regenerated and the 3D model of the gear obtained.

**TOOTH CONTACT ANALYSIS**

After regeneration of the gear tooth surfaces from the point clouds, tooth contact analysis can be applied. The goal of the application of tooth contact analysis is to verify the contact pattern on the gear tooth surfaces, discover the existence of micro-geometry modifications, and evaluate the unloaded function of transmission errors. Three different regular grids have been used to regenerate the gear tooth surfaces: (i) a regular grid of 15 x 10 points as shown in Figure 3(a), (ii) a regular grid of 30 x 20 points as shown in Figure 3(b), and (iii) a regular grid of 50 x 30 points as shown in Figure 3(c).

Figure 7 shows the comparison of the contact patterns obtained when considering the mesh of Gear A (left) and Gear B (right) with a master gear of 34 teeth. The density of the grid to regenerate the gear tooth surfaces is of 15 points in longitudinal direction and 10 points in profile direction. Figure 7 shows a clear difference between the quality of the gear tooth surfaces of Gear A and Gear B. Gear A shows a regular contact pattern and the contact area is extended over the whole gear tooth surface. However, for Gear B, certain deviations make the contact pattern not cover completely the active surface of the gear tooth. There is also a clear difference in the peak-to-peak level of transmission errors in favor of Gear A as the gear with better quality.

Figure 8 shows the comparison of the contact patterns for Gear A and Gear B.
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Figure 7: Contact pattern and function of transmission errors for Gear A (left) and Gear B (right) in mesh with a master gear and regenerated by using a regular grid of 15 x 10 points.

Figure 8: Contact pattern and function of transmission errors for Gear A (left) and Gear B (right) in mesh with a master gear and regenerated by using a regular grid of 30 x 20 points.

Figure 9: Contact pattern and function of transmission errors for Gear A (left) and Gear B (right) in mesh with a master gear and regenerated by using a regular grid of 50 x 30 points.

Figure 10: Discrete Fourier Transform (DFT) of the function of transmission errors. Left column represents Gear A and right column represents Gear B. From top to bottom: grid 15 x 10, grid 30 x 20, and grid 50 x 30.

Figure 11: Illustration of additional functions $d_1$ to $d_4$ used to adjust the boundaries of the reverse engineered geometry (theoretical geometry).

(left) and Gear B (right) in mesh with a master gear when a grid density of 30 x 20 points is used for regeneration of gear tooth surfaces. The function of transmission errors is very sensitive to the grid density used for reconstruction of NURBS surfaces from the point cloud. However, from a general point of view, the better quality of Gear A is also observed, not only with regards to the contact pattern but also considering the peak-to-peak values of the function of transmission errors.

Finally, Figure 9 shows similar results when a grid of 50 x 30 points is used for reconstruction of the gear tooth surfaces. Again, comparing the results obtained by considering similar grid densities, the claim that the quality of Gear A is better than Gear B is again consistent with the obtained results.

It is widely accepted that the amplitude spectrum of the function of transmission errors will excite vibrations in the gear drive that will be radiated as noise, so that for lower amplitude spectrum of the function of transmission errors, lower excitation of vibrations will occur and therefore, lower gear whine noise might be expected [3]. Figure 10 shows the amplitude spectrum of the functions of transmission errors shown above in Figures 7, 8, and 9 for Gears A and B considering various densities of the regular grid used for regeneration of gear tooth surfaces. A speed of 2000 rpm has been considered for the pinion and therefore the meshing frequency for a gear of 20 teeth is of 666.67 Hz.

Gear A shows systematically lower amplitude spectrum of the function of transmission errors and therefore Gear A is expected to radiate lower levels of noise and vibration. The values of the amplitudes are highly sensitive to the density of the grid used for regeneration of gear tooth surfaces from point clouds but the comparison between Gear A and Gear B considering same densities of the regular grid will consistently yield same conclusion: Gear A will radiate less noise and vibration than Gear B.

REVERSE ENGINEERING OF GEAR GEOMETRY

Noncontact metrology allows reverse engineering of gear geometry to be performed fast and accurately. No matter what the number of design parameters is, an optimization algorithm that minimizes the deviation between a theoretical geometry and the regenerated surfaces from point clouds will give us the closest geometry to the point cloud in few seconds. An approach for reverse engineering of gear parameters and gear tooth proportions of helical gears has been developed and the obtained results shown below.

The preliminary known geometric parameters are:

- **Number of teeth**: 20 teeth (from direct inspection of the data).
- **Hand of helix**: Right (from direct inspection of the data).
- **Tentative normal module**: 3 mm. The tentative normal module is obtained using the addendum and root radius shown in Figure 2. Considering that a standard tooth geometry has a whole depth equal to 2.25 times the module, the tentative normal module of the gear shown in Figure 2 can be obtained as follows:

$$m_n = \frac{r_{\text{add}} - r_{\text{root}}}{2.25} = \frac{34.91 - 27.90}{2.25} = 3.1156 \Rightarrow 3.0 \text{ mm}$$

The following seven variables have been used for the process of reverse engineering:

- Pressure angle.
- Helix angle.
- Face width.
- Addendum coefficient.
- Dedendum coefficient.
- Profile shift coefficient.
- Edge radius coefficient.

Not all the variables considered are needed for the reverse engineering of the helical gear geometry. For example, the face width could be considered as known but it was considered as a variable here to test the automatic adjustment of the limits of the to-be-obtained geometry to those of the objective geometry. This is particularly important to reverse engineer the geometry of bevel gears. The objective function to be minimized consists of the sum of the squared value of the surface...
deviations between a theoretical and the objective geometry on a grid of points on the contacting surfaces plus four additional distances $d_1$ to $d_4$ to adjust the boundaries of the surfaces in the radial projection (2D) of the tooth surfaces, as shown in Figure 11. With it, the addendum and dedendum coefficients as well as the face width of the helical gear are obtained.

Table 1 shows the results of the process of reverse engineering of the helical gear geometry. The initial value of each parameter used in the search, the minimum and maximum values that were allowed for each variable and the computed values are shown. The Levenberg-Marquardt algorithm (LM) with a trust-region strategy has been applied here. It consists of an iterative algorithm used in nonlinearly bound-constrained and unconstrained optimization problems in which the objective function is formulated in terms of least-squares. The LM algorithm belongs to the family of second derivative unconstrained methods and constitutes an improvement of Newton’s method for nonlinear least-squares problems [5].

Based on the obtained results shown in Table 1, there is no doubt that the intended pressure angle was 20.0 degrees, the intended helix angle was of 20 degrees, and the gear was manufactured with a positive profile shift coefficient of 0.054615. The value of the edge radius coefficient shown in Table 1 converges to the lower minimum value because the boundaries of the fillet surface and not the deviations all over the fillet surface were included into the objective function being minimized. Deviations from the fillet surface should be included into the optimization algorithm to get a better approximation of the edge radius coefficient. Also, the obtained values for the face width and the addendum coefficient are influenced by the boundary drawn by the user and shown in Figure 2. Those values can be adjusted to match the desired values if the gear is to be replaced by a new manufactured gear. Moreover, it is clear from the obtained results shown in Table 1 that the gear was generated with a long dedendum coefficient of 1.4. Additional design parameters can be added to the outlined reverse engineering process of helical gears if needed to match any micro-geometry modification or specific manufacturing processes.

Figure 12 shows the comparison of the reverse engineered helical gear geometry and the objective geometry regenerated from a point cloud.

Table 1: Results of reverse engineering of the helical gear geometry.

<table>
<thead>
<tr>
<th>Gear Parameter</th>
<th>Initial Value</th>
<th>Min/Max Values</th>
<th>Computed Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure angle, $\alpha$</td>
<td>22.5°</td>
<td>19.5° to 25.5°</td>
<td>0.349029 rad [19.9979°]</td>
</tr>
<tr>
<td>Helix angle, $\beta$</td>
<td>22.5°</td>
<td>10.0° to 10.6°</td>
<td>0.349080 rad [19.9997°]</td>
</tr>
<tr>
<td>Face width, $b_w$</td>
<td>20.0 mm</td>
<td>20.0 mm / 40.0 mm</td>
<td>27.342657 mm</td>
</tr>
<tr>
<td>Addendum coefficient</td>
<td>1.0</td>
<td>0.5 / 1.5</td>
<td>0.914994</td>
</tr>
<tr>
<td>Dedendum coefficient</td>
<td>1.0</td>
<td>0.5 / 1.5</td>
<td>1.387717</td>
</tr>
<tr>
<td>Profile shift coefficient, $x$</td>
<td>0.0</td>
<td>-1.0 / 1.0</td>
<td>0.054615</td>
</tr>
<tr>
<td>Root radius coefficient, $\rho$</td>
<td>0.25</td>
<td>0.15 / 0.40</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Figure 13: Finite element model of a helical gear set with five pairs of contacting teeth.

Figure 14: Contact pressure representation on the pinion tooth surfaces for three different contact positions: contact point 1 (left), contact point 21 (center), and contact point 41 (right).
FINITE ELEMENT ANALYSIS

Finite element models comprising five pairs of contacting teeth were considered for finite element analysis. The solver of ABAQUS computer program was used to perform the analysis. The models were generated automatically from the Integrated Gear Design (IGD) software and therefore all nodes on the finite element mesh are obtained as points of the gear tooth surface, avoiding the loss of accuracy when the finite element models are built with intermediate CAD models. The model size consists of 190,800 elements and 224,666 nodes. Figure 13 shows one of the finite element models that have been used for the analyses. Three-dimensional solid elements of type C3D8I [6] have been used, being hexahedral first order elements enhanced by incompatible deformation modes in order to improve their bending behavior [6]. The pinion and gear materials are defined as steel with elastic modulus of 210 GPa and Poisson ratio of 0.3.

A torque of 350 N·m has been applied to the pinion of the gear set for all cases investigated. Firstly, the finite element analysis of the reverse engineered pinion from point clouds in mesh with a master gear of 34 teeth will be performed with and without considering tip relief. The results obtained will be labeled as “Master” and “Master TR” for the case without tip relief and with tip relief, respectively. The contacting surfaces of these gears are theoretical, and the maximum stresses obtained can be considered as reference level for the considered geometry. The application of tip relief in helical gears can improve gear performance by preventing contact near the leading edge of the teeth throughout a cycle of meshing. In helical gears, contact at the tip of the tooth is maintained almost throughout the entire cycle of meshing, making tip relief modifications particularly important for this type of gear. The results that will be shown below will consider the contact pressure and maximum bending stress on the middle tooth of the pinion (Teeth #3) of the finite element model of 5 pairs of contacting teeth along 41 contact positions covering two cycles of meshing of the gear set.

Figure 14 shows the patterns of contact pressure on the pinion tooth surfaces for three different contact positions: 1, 21, and 41. The contact line crosses the top edge boundary of the gear tooth surfaces of Teeth #3 from contact point 23 until contact point 41, and therefore the effect of edge contact is clearly seen in the following figures showing the evolution of the contact pressure.

Figures 15, 16, and 17 show the evolution of maximum contact pressure (left) and the maximum bending stress (right) for the middle tooth of the finite element model (Teeth #3) for a reverse engineered master pinion without tip relief (Master), reverse engineered master pinion with tip relief (Master TR), regenerated Gear A, and regenerated Gear B, for different regular grid sizes varying from 15 x 10 points for Figure 15, 30 x 20 points for Figure 16, and 50 x 30 points for Figure 17.

From Figures 15 to 17, notice that the maximum bending stresses are not sensitive to the grid size considered for definition of the regenerated tooth surfaces of Gears A and B. Gear B consistently shows higher bending stresses than Gear A. It is also worth it to notice that the master pinion with tip relief shows higher bending stresses than the master pinion without tip relief. However, application of tip relief is essential for reducing the maximum contact stresses and contact pressure. The results obtained for the maximum contact pressure for all sizes of the regular grid are very sensitive to the size of the regular grid, and the results show high fluctuations along the 41 contact positions due to the augmented influence of the surface deviations on contact pressure and contact stresses obtained by the finite element method.

From the point of view of contact pressure and bending stresses, Gear A shows lower levels of contact pressures and bending stresses. Although contact pressure is very sensitive to the grid size used for surface regeneration, the bending stresses are very consistent across all grid sizes.

CONCLUSIONS AND FUTURE PERSPECTIVE

Based on the performed research, the following conclusions can be drawn:

- The analysis and simulation of gear drives using regenerated gear tooth surfaces from point clouds is fast and may become a valuable tool to evaluate the mechanical performance of different versions or different batches of gears.
- Regular grids of equal sizes should be used to compare the mechanical behavior of gears when their surfaces are regenerated from point clouds. The function of transmission errors, the amplitude spectrum of the function of transmission errors and the maximum contact pressure are very sensitive to the size of the regular grid used for the regeneration of the contacting surfaces.
- Increasing the number of points used for the regular grid that defines the best fit NURBS surface does not yield better results than grids of lower number of nodes. If the regular grid is too dense, the NURBS surface amplifies certain irregularities of the gear tooth surfaces and does not contribute to obtain better results from TCA
Finite element models created by regenerated gear tooth surfaces from point clouds yield highly oscillating results in terms of contact pressure on the active surfaces of the gears. However, the maximum bending stress can be evaluated effectively, and its maximum value does not depend on the grid size used for regeneration of the gear tooth surfaces. This information is very valuable to be used as a baseline for further improvement and optimization of existing gear drives.

The presented approach for integration of non-contact metrology in the process of analysis and simulation of gear drives has shown promising results and provides added value for the virtual testing and simulation of gear drives. However, further research work has to be carried out to improve the results of simulation. Filtering and denoising the point clouds have been shown necessary before the gear tooth surfaces are regenerated using the presented approach.

Non-contact metrology provides a wealth of data to perform very accurate reverse engineering of geometric parameters of gear drives.

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